



## Modelling and Solving Problems of Sustainable Efficiency of Technological Processes in Metallurgy

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<p>Received: <i>January 20, 2026</i> Peer-reviewed: <i>February 25, 2026</i> Accepted: <i>May 4, 2026</i></p>	<p><b>ABSTRACT</b> Through a comparative analysis, this study investigates the development, simulation, and application of mathematical models for integrated analysis of copper rod production in order to improve product quality, reduce costs, and minimize risks. The purpose of this article is achieved by creating mathematical descriptions of real processes, which are then used to conduct computer experiments. In contrast to the traditional molecular dynamic methods, this study used information technology to obtain the characteristics of the flow field in technological equipment. The novelty lies in: integration of thermomechanical modeling with an optimization algorithm; introduction of a criterion minimizing mechanical property variation; consideration of reduction and redistribution effects on roll wear; possibility of adaptive real-time control. An information analysis of the profiles of copper melt flow velocities in a wide range of temperatures in the Copper Rod Production Plant Kazkat is presented. Information Technology fulfils the need to address the problem of determining the optimal values of temperature, rolling speed, and other parameters to achieve the best quality and productivity of copper rod production processes. Based on these findings, this study proposes the optimization directions for the temperature field profile in the rolling rolls, which leads to a decrease in roll wear and an increase in the uniformity of the wire rod structure, focusing on improving microstructural properties.</p>
	<p><b>Keywords:</b> simulation, copper, computer modelling, melt, information technology, numerical method.</p>
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### Introduction

The economic growth of Kazakhstan is providing local enterprises with the necessary goods and materials produced at the national level that meet high-quality standards and scientific algorithms for theoretical research. In the mining and metallurgical industry, there is a need to create high-quality products that meet the needs of the market, as well as the creation of scientific algorithms for the study of metals with different physical properties and inhomogeneities. Simulations of technological processes in metallurgical production, such as process modeling in metallurgical production, is

aimed at solving production problems with molten systems for approaching sustainable efficiency.

Contribution [1] combines molecular dynamic methods, the properties of AI, and the matrix composite.

This study presents a new technology for low-carbon hydrogen metallurgy based on hydrogen instead of carbon and a method for producing pig iron for a reducing smelting furnace [2].

This article focuses on the simulations of technological processes in metallurgical production. The study presents a new technology associated with copper staves. The method of mathematical modelling and analysis of slag crust in the bosh was conducted [3]. The solutions to numerous problems

and technical indicators associated with the copper slag crust were investigated.

This study is concerned with microstructure evolution in high stacking fault metals and alloys, «it is relevant to the high-temperature extrusion of aluminum alloys» [4].

Recommendations are proposed for technological optimization and the use of technologies to create an integrated low-carbon metallurgical system, storage technologies, and carbon cycling technologies [5].

The purpose of this article is to provide local enterprises with the necessary goods and materials produced at the national level that meet high-quality standards and scientific algorithms for theoretical research for the production of «titanium sponge from concentrates of the Satbaevskoye deposit in Kazakhstan. The article uses an innovative approach for obtaining titanium slag from low-grade Kazakhstan ilmenite concentrates characterized by a high content of refractory components» [6].

To experiment, the water model of the oxygen with recommendations for technological optimization and the use of technologies to create an integrated metallurgical system, storage technologies, carbon cycling technologies, and the effects of a new retaining wall device are proposed for innovative research [7].

One of the most resource-intensive and strategically important industries is the production of copper rods, which have various physical and chemical characteristics. This trend is the demand of society for high-quality materials with high-performance properties. To solve the corresponding technological problems, it is necessary to use mathematical modelling tools and informatization, cognitive tools, which make it possible to study the static or dynamic characteristics of the object under study [[8], [9]]. The purpose of this article is to simulate technological processes in metallurgical production, as process modeling in metallurgical production is aimed at solving production problems with molten systems for approaching sustainable efficiency.

To experiment with recommendations for technological optimization and the use of technologies to create an integrated metallurgical system, technologies for creating retaining walls for innovative research are proposed for the development of the metallurgical industry [10]. Modeling of technological processes in metallurgical production is aimed at solving production problems using various microstructures to achieve sustainable efficiency [11].

The solution of production problems at modern metallurgical enterprises is carried out using the Recommendation for Technological Optimization, aimed at an analytical solution based on differential equations proposed in [12].

Recommendations are offered on the use of technologies for the purpose of technological optimization of production, storage technologies, cycling technologies, modelling [[13], [14], [15], [16], [17], [18]].

### Theoretical framework

Particular attention should be paid to intensifying work on the creation of a new copper smelter that meets modern environmental requirements [[19], [20]]. The importance of the metallurgical industry and non-ferrous metallurgy for the economy at present can hardly be overestimated - the bulk of all spheres of production enjoy the results of metallurgical production. Metallurgy is of great importance for the development of national production and ensuring the modernization of the economies of countries, their integration into the world economy [[21], [22], [23], [24], [25], [26]].

The above technologies use high-quality scientific and technical problems [[27], [28]]. The study of various areas made it possible to design the most optimal option for cognitive studies on intellectual results [[29], [30], [31]].

There is a movement of convective flows or thermodynamic flows, where the bulk force divides the flow into the bottom layer and the main flow. The main flow moves upwards, and the denser lower layer descends, creating circulation and mixing of the medium, as when heated from below [[32], [33], [34]].

Mathematical modelling is a powerful tool for achieving a competitive advantage in the production of copper rods. Kazakhstan has significant copper reserves, which makes the production and export of copper rod a key area for the development of the mining industry. This study fulfils the need for determining the optimal values of temperature, rolling speed, and other parameters to achieve the best quality and productivity of copper rod production processes. Mathematical modelling of the copper rod process aims to optimize the temperature field profile, which leads to an increase in the uniformity of the wire rod structure at the Copper Rod Production Plant Kazkat. The production of copper at modern metallurgical enterprises is

carried out using the following technologies: SCR, SMS Contirod, UPCAST Outokumpu.

The production scheme of copper rods is presented in Figure 1.

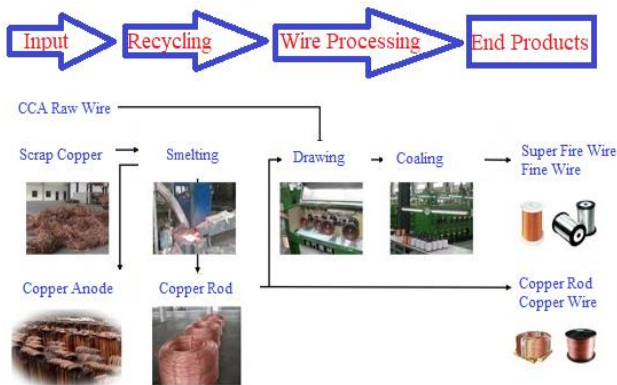


Figure 1 - Production scheme of copper rods

At present, the copper industry in the country is represented by the production of copper concentrate products. Half of the exported materials fall on the share of ore concentrates and metals, which occupy the most important place in the list of products exported from the country [[27], [35], [36]].

It is well known that the approximation of hydrodynamic equations leads to nonlinear systems of algebraic equations. Consequently, their numerical solution is associated with significant computational difficulties. In this regard, various approaches to the construction of operator-splitting schemes for the Navier–Stokes equations in the sense of weak approximation were considered.

Let us outline the general principle for constructing splitting schemes for the Navier–Stokes equations. In a bounded domain  $\Omega \in R^3$ , we consider a system of nonlinear stationary equations subject to prescribed boundary conditions. In addition, a temperature model of a heterogeneous melt is formulated with specified initial and boundary conditions.

Monitoring and analyzing melt motion constitute the central problem of this study. It is assumed that the external forces acting on the melt under boundary conditions are known, and that the initial velocity field is specified in the case of unsteady flow. The physical and mathematical model of melt motion is formulated in a coordinate system in which the computational domain associated with the melt remains fixed.

The metal melt moved along the inclined chute of the metallurgical equipment. This study aims to

address obtaining high-quality products with ever-increasing demand from machine builders while minimizing the costs of their production. The modelling framework is based on the fundamental principles of continuum mechanics and thermodynamics. The motion of the molten copper is described using the Navier–Stokes equations for incompressible viscous flow (1)-(3):

Continuity equation:

$$\nabla \cdot \mathcal{G} = 0 \tag{1}$$

Momentum conservation equation:

$$\rho \left( \frac{\partial \mathcal{G}}{\partial t} + (\mathcal{G} \cdot \nabla) \mathcal{G} \right) = \mu \nabla^2 \mathcal{G} - \nabla p + \rho f \tag{2}$$

Energy equation (with thermal conductivity)

$$\rho c_p \left( \frac{\partial T}{\partial t} + \mathcal{G} \cdot \nabla T \right) = \theta \nabla^2 T \tag{3}$$

For steady-state flow conditions:

$$\frac{\partial \mathcal{G}}{\partial t} = 0$$

with initial boundary conditions:

$$\mathcal{G}|_{t=0} = \mathcal{G}_0(x), t|_{t=0} = t_0(x), \mathcal{G}|_S = 0, t|_S = 0, \tag{4}$$

where:

$\mathcal{G}$  — velocity vector of the melt;

$\rho$  — density;

$p$  — pressure;

$\mu$  — dynamic viscosity;

$f$  — gravitational acceleration vector;

$c_p$  — specific heat capacity;

$\theta$  — thermal conductivity.

Let the metal melt move along the inclined chute of metallurgical equipment. The physical and mathematical model of this technological process is built under the assumption that the length of the trough is infinite, the metal melt moves along the axis of the trough in such a way that the velocity function  $\omega$  depends only on the variable  $x, y$  whereas, the pressure function depends on the variable  $z$ . Such motions are called steady-state motions. As a result, we obtain a model with isothermal motion of the melt, in which density  $\rho$  and viscosity  $\mu$  are constant. Hence, the metal melt moves along the inclined chute of metallurgical equipment, as given in Navier-Stokes equations form (5). The change in pressure is negligible from

section to section, keeping the same value in a given section. Such motions are called steady-state motions. Thus, based on (5), we obtain the following equation (6).

$$\begin{cases} -\frac{1}{\rho} \frac{\partial p}{\partial x} = 0, \\ -\frac{1}{\rho} \frac{\partial p}{\partial y} = 0, \\ \omega \frac{\partial \omega}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \xi \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^2 \omega}{\partial z^2} \right), \\ \frac{\partial \omega}{\partial z} = 0. \end{cases} \quad (5)$$

$$\frac{dp}{dz} = \lambda \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right). \quad (6)$$

The right side of (6) depends of  $x, y$  coordinates, the left side depends of  $z$  coordinate. Let us the main statements of hydrodynamics:

$$\frac{dp}{dz} = -\frac{\Delta p}{\lambda},$$

$\lambda$  is the chute length.

When the melt flows through an inclined trough, there is a movement of convective flows or thermodynamic flows, where the bulk force divides the flow into the bottom layer and the main flow. Internal friction forces slow down the movement of the atoms of the lower layer. This deceleration is transferred from one layer to another along the entire flow to the surface of the copper melt; there is a free surface of the metal melt. So, the pressure will be equal to the atmospheric pressure.

This study aims to address the angle of inclination, which is equal to  $\Theta$ , a volumetric force, and is equal to  $F_z = \lambda \sin \Theta = \frac{\Delta p}{\ell}$ . So

$$\lambda \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) + \rho \lambda \sin \Theta = 0. \quad (7)$$

$$\omega = 0 \quad y = 0, \quad \frac{\partial \omega}{\partial y} = 0, \quad y = \lambda_1, \quad \frac{\partial \omega}{\partial x} = 0, \quad x = \lambda_2. \quad (8)$$

Calculations were constructed for industrial equipment at the Copper Rod Production Plant Kazkat. We obtained data from industrial partners or public repositories.

Production of copper rods requires a constant search for technical solutions aimed at reducing harmful impurities and oxygen to the lowest

possible concentrations. The study of various areas made it possible to design the most optimal option for copper rods using modern Continuous Casting and Rolling Technology. We conduct cognitive studies on intellectual results with existing measures of copper melt velocities. As a result, the problem of decision-making is solved using the proposed cognitive measure of similarity to check its applicability.

Modern achievements in this area have led to the production of high-quality copper at the Copper Rod Production Plant Kazkat, in Figure 2. The technological scheme of the equipment is shown in Figure 3.

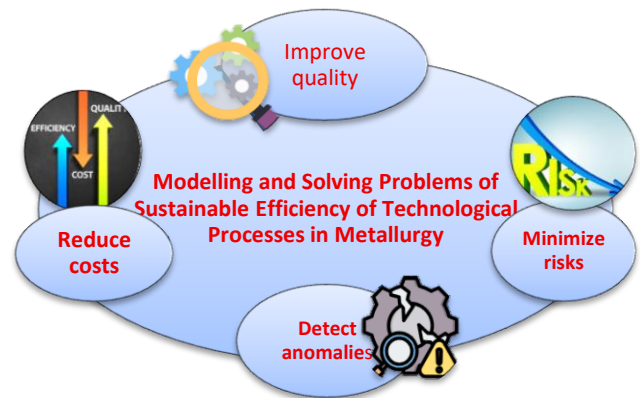


Figure 2 - Theoretical framework

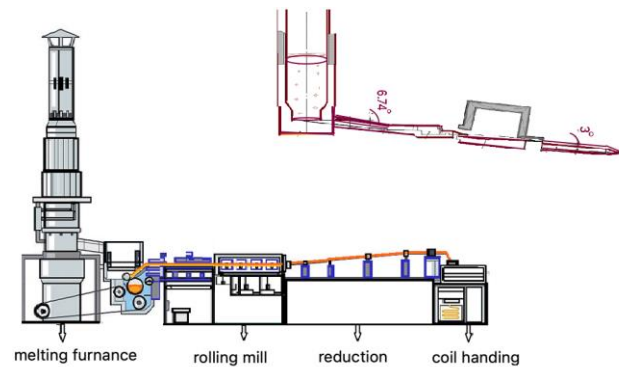


Figure 3 - Technological scheme of SCR Plant Kazkat, measurements are given in [mm]

### Numerical Modelling

Using Equations (9) – (11), we calculated the numerical values of the melt flow parameters for the Copper Rod Production Plant Kazkat business perimeter for the lower chute with an inclination angle of  $3^\circ$ , as shown in Fig. 4:

$$S = \frac{[\lambda r - \sigma(r-h)]}{2}, \quad (9)$$

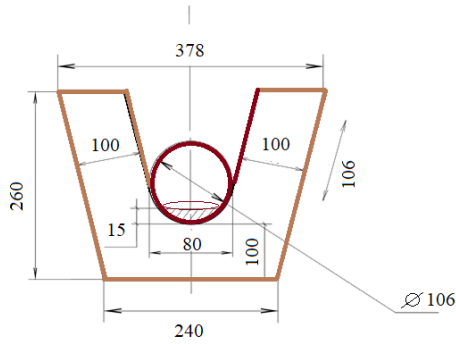
$\lambda$  is arc length,  $\sigma$  is chord,  $h$  – is segment arrow:

$$\sigma = 80[\text{mm}], \quad h = 16[\text{mm}],$$

$$\lambda \approx \sqrt{\sigma^2 + \left(\frac{15h^2}{3}\right)} = \sqrt{80^2 + \left(15 \cdot \frac{16^2}{3}\right)} = 87.6[\text{mm}^2]. \quad (10)$$

$$S = \frac{\left[87,6 \cdot \frac{104}{2} - 80 \left(\frac{104}{2} - 16\right)\right]}{2} = 1675.2[\text{mm}^2]. \quad (11)$$

The average flow velocity of the copper melt is equal to  $v_{middle} = 0.42 \left[\frac{\text{m}}{\text{s}}\right]$ .



**Figure 4** - Section of lower chute (measurements given in [mm])

The governing Navier–Stokes equations (5)–(6) were discretized using the finite volume approach.

$$\frac{\partial}{\partial t} \int_V \rho \phi dV + \left[ \int_S \rho \phi \vec{u} \cdot \vec{n} dS \right] = \left[ \int_S \Gamma \nabla \phi \cdot \vec{n} dS \right] + \int_V S_\phi dV \quad (12)$$

$\phi$  is a general transported variable (e.g., velocity component or energy),

$\Gamma$  is the diffusion coefficient,

$S_\phi$  is the source term,

$\vec{n}$  is the outward unit normal vector on the surface.

Spatial discretization of the convective terms was performed using a second-order upwind scheme (13) to achieve higher-order accuracy and reduce numerical diffusion. Time integration was performed using a second-order implicit scheme (14), which ensures enhanced temporal accuracy and improved stability characteristics of the numerical solution.

$$\phi_f = \phi_p + \nabla \phi_p \cdot (\vec{r}_f - \vec{r}_p), \quad (13)$$

$P$  denotes the upstream control volume,  $\vec{r}_f, \vec{r}_p$  are position vectors of the face center and cell center, respectively.

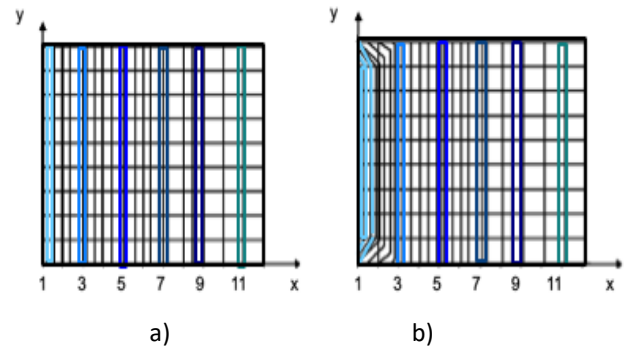
The transient term is discretized using the second-order backward differencing formula (BDF2):

$$\left(\frac{\partial \phi}{\partial t}\right)^{n+1} = \frac{3\phi^{n+1} - 4\phi^n + \phi^{n-1}}{2\Delta t}, \quad (14)$$

which yields second-order temporal accuracy  $O(\Delta t^2)$ .

The scheme is implicit because the unknown value  $\phi^{n+1}$  appears in the discretized equation and is obtained by solving the resulting algebraic system.

In the calculations of the numerical scheme  $\Delta x = \Delta y = 0,02$  we used constant time step sizes  $\Delta t = 0,001$ . Calculated profiles of flow velocities  $v$  and  $u$  are shown in Fig. 5.



**Figure 5** - (a) transverse  $v$  and (b) longitudinal  $u$  melt flow velocity profiles

A structured computational grid with local refinement in the near-wall regions was employed to ensure accurate resolution of boundary layer phenomena. The mesh was selectively refined in areas with high velocity and pressure gradients to improve predictions of wall shear stresses and heat transfer, while maintaining a reasonable overall cell count and computational cost. A structured grid with local refinement near walls was used.

Three grid levels were tested:

- $1.2 \times 10^5$  cells
- $3.8 \times 10^5$  cells
- $9.5 \times 10^5$  cells

Grid independence was achieved when the variation in mean velocity was below 2%.

The simulation and application of mathematical parameters include movement of convective flows or thermodynamic flows, the bulk force, and internal friction forces when the melt flows in an inclined trough.

The temperature dependence of the dynamic viscosity was described by the Arrhenius-type equation:

$$\mu_{model}(T) = A \exp\left(\frac{Q}{RT}\right) \quad (15)$$

$$A = 3.2 \times 10^{-4} \text{Pa}\cdot\text{s}$$

$$Q = 39,000 \text{J/mol}$$

$$R = 8.314 \text{J/(mol}\cdot\text{K)}$$

The strength of the relationship between the experimental data and the values calculated using (15) was evaluated using the coefficient of nonlinear multiple correlation (16), together with an assessment of its statistical significance using the Fisher criterion (17).

The coefficient of nonlinear multiple correlation was determined as:

$$R = \sqrt{1 - \frac{(n-1) \sum_{i=1}^n (y_{\text{exp},i} - y_{\text{calc},i})^2}{(n-k-1) \sum_{i=1}^n (y_{\text{exp},i} - \bar{y}_{\text{exp}})^2}} \quad (16)$$

$$F = \frac{R^2}{1-R^2} \frac{n-m-1}{m}, \quad (17)$$

$y_{\text{exp},i}$  experimental value

$y_{\text{calc},i}$  calculated value,

$\bar{y}_{\text{exp},i}$  mean experimental value,

$n$  number of experimental data points,

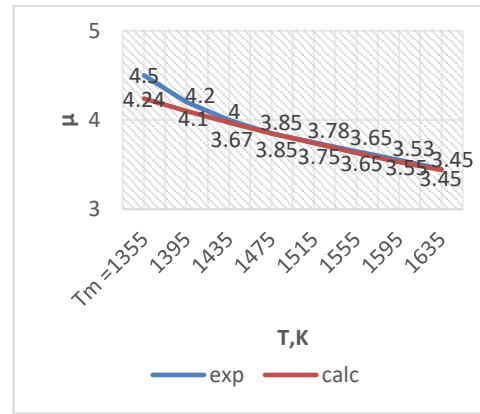
$m$  number of independent variables in the regression model.

Let us construct an algorithm for calculating the viscosity of a copper melt and establish the dependence of viscosity on temperature. The results are presented in Table 1 and Figure 6.

**Table 1** - Calculated and experimental data [37] of the viscosity

$T$	$\mu_{\text{exp}}$	$\mu_{\text{calc}}$	$\Delta$ (model-exp)	Relative Error (%)
$T_m=1355$	4.5	4.24	-0.26	-5.8
1395	4.2	4.10	-0.10	-2.4
1435	4.0	3.97	-0.03	-0.8
1475	3.5	3.85	-0.00	0.00
1515	3.75	3.74	-0.01	-0.03
1555	3.65	3.63	-0.02	-0.05
1595	3.55	3.53	-0.02	-0.06
1635	3.45	3.44	-0.01	-0.03
MAPE	-	-		1.34

The correlation coefficient  $R=0.986626$  is high, therefore, and shows the correctness and adequacy of our verification of theoretical results.



**Figure 6** – The copper viscosity as a function of temperature

Comparison between the proposed model and experimental viscosity data of liquid copper in the temperature range 1355–1635 K demonstrates excellent agreement,  $R^2 = 0.91$ , MAPE = 1.34%. The maximum deviation 5.8% occurs near the melting temperature, likely due to structural fluctuations and the assumption of constant activation energy. In the range 1435–1635 K, the model accuracy remains within 1%, confirming its suitability for engineering calculations.

The coefficient of determination is calculated using the standard formula for model validation:

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}}$$

$SS_{\text{res}} = \sum (\mu_{\text{exp}} - \bar{\mu}_{\text{model}})^2$  is the sum of squared residuals,

$SS_{\text{tot}} = \sum (\mu_{\text{exp}} - \bar{\mu}_{\text{exp}})^2$  is the total sum of squares, representing the total variance of the experimental data.

$\bar{\mu}_{\text{exp}}$  is the mean experimental viscosity.

$R$	$R^2$	$SS_{\text{res}}$	$SS_{\text{tot}}$	$\bar{\mu}_{\text{exp}}$
0.986626	0.91	0.0795	0.842	3.869mPA

## Results and Discussion

The copper flow velocity distribution profiles at 1355 K, 1395 K, 1435 K, 1475 K, 1515 K, 1555 K, 1595 K, and 1635 K are presented in Figures 7 – 14 and Tables 2-9.

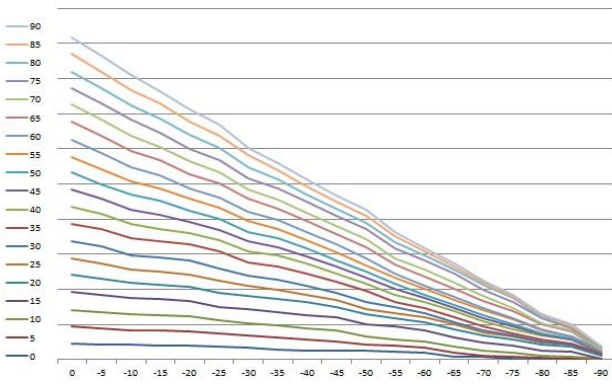


Figure 7 - Velocity isolines at 1355 K

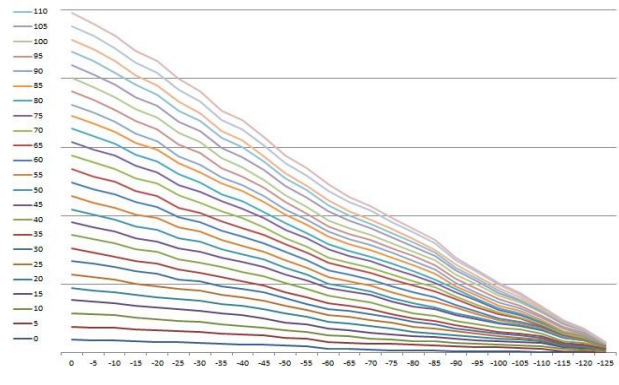


Figure 11 - Velocity isolines at 1515 K

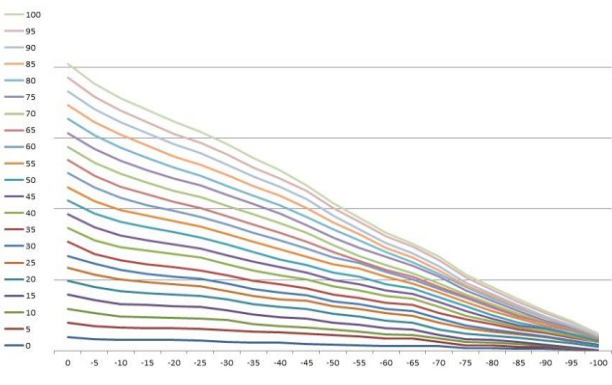


Figure 8 - Velocity isolines at 1395 K

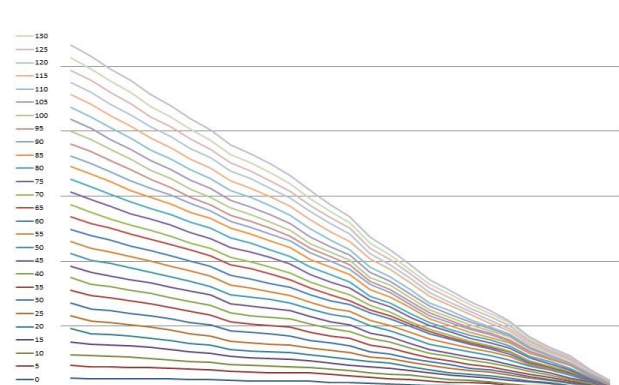


Figure 12 - Velocity isolines at 1555 K

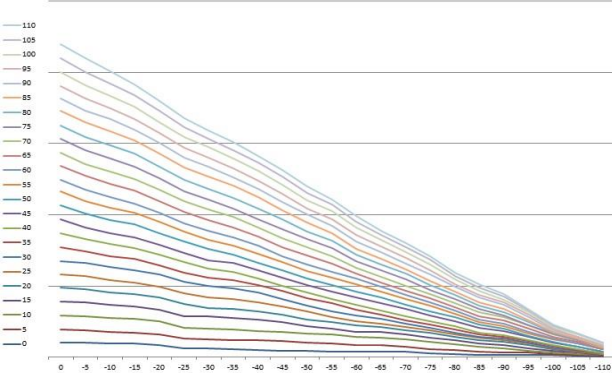


Figure 9 - Velocity isolines at 1435 K

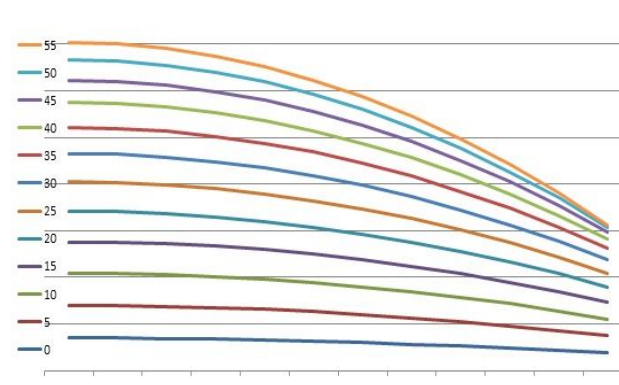


Figure 13 - Velocity isolines at 1595 K

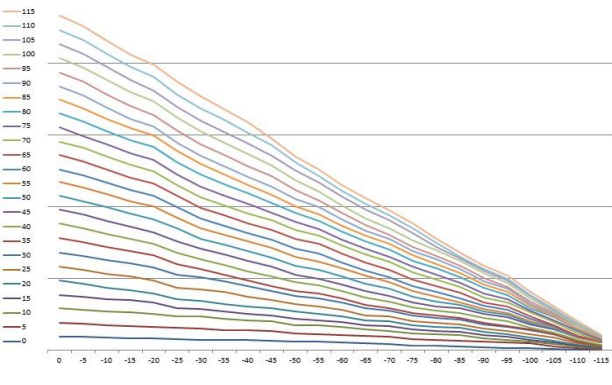


Figure 10 - Velocity isolines at 1475 K

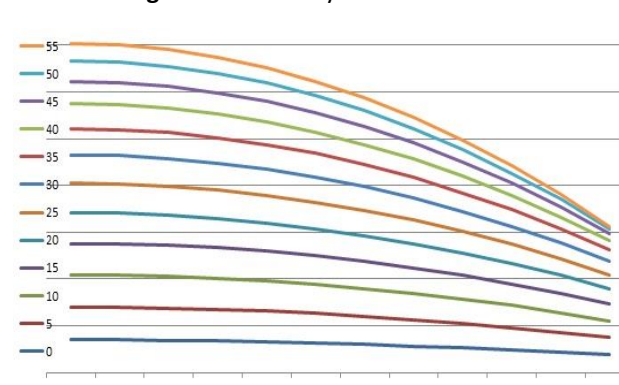


Figure 14 - Velocity isolines at 1635 K

**Table 2** - Data from numerical calculations of the copper melt motion at 1355 K

x \ y	0.00	-10.00	-20.00	-30.00	-40.00	-50.00
0.00	0.4021	0.3963	0.3774	0.3461	0.3032	0.2471
10.00	0.3932	0.3900	0.3722	0.3400	0.2972	0.2412
20.00	0.3773	0.3711	0.3520	0.3210	0.2783	0.2223
30.00	0.3461	0.3402	0.3213	0.2913	0.2471	0.1923
40.00	0.3030	0.2973	0.2783	0.2470	0.2051	0.1484
50.00	0.2472	0.2410	0.2220	0.1910	0.1480	0.0922

**Table 3** - Data from numerical calculations of the copper melt motion at 1395 K

x \ y	0.00	-10.00	-20.00	-30.00	-40.00	-50.00
0.00	0.4422	0.4342	0.4142	0.3801	0.3324	0.2711
10.00	0.4341	0.4280	0.4073	0.3732	0.3250	0.2642
20.00	0.4142	0.4070	0.3872	0.3520	0.3053	0.2442
30.00	0.3801	0.3730	0.3523	0.3190	0.2710	0.2105
40.00	0.3320	0.3250	0.3051	0.2712	0.2243	0.1623
50.00	0.2710	0.2640	0.2440	0.2101	0.1620	0.1023

**Table 4** - Data from numerical calculations of the copper melt motion at 1435 K

x \ y	0.00	-10.00	-20.00	-30.00	-40.00	-50.00
0.00	0.4823	0.4742	0.4511	0.4142	0.3620	0.2962
10.00	0.4740	0.4670	0.4440	0.4072	0.3555	0.2880
20.00	0.4513	0.4440	0.4232	0.3851	0.3330	0.2660
30.00	0.4140	0.4070	0.3850	0.3490	0.2964	0.2290
40.00	0.3620	0.3550	0.3333	0.2960	0.2442	0.1770
50.00	0.2962	0.2883	0.2660	0.2290	0.1770	0.1120

**Table 5** - Data from numerical calculations of the copper melt motion at 1475 K

x \ y	0.00	-10.00	-20.00	-30.00	-40.00	-50.00
0.00	0.5231	0.5137	0.4901	0.4500	0.3940	0.3210
10.00	0.5140	0.5070	0.4820	0.4420	0.3859	0.3131
20.00	0.4900	0.4820	0.4590	0.4181	0.3610	0.2890
30.00	0.4500	0.4420	0.4178	0.3780	0.3210	0.2490
40.00	0.3939	0.3860	0.3610	0.3213	0.2660	0.1920
50.00	0.3209	0.3128	0.2890	0.2490	0.1920	0.1210

**Table 6** - Data from numerical calculations of the copper melt motion at 1515 K

x \ y	0.00	-10.00	-20.00	-30.00	-40.00	-50.00
0.00	0.5659	0.5630	0.5300	0.4870	0.4260	0.3470
10.00	0.5560	0.5550	0.5210	0.4780	0.4170	0.3390
20.00	0.5301	0.5280	0.4958	0.4522	0.3910	0.3129
30.00	0.4870	0.4850	0.4520	0.4091	0.3469	0.2690
40.00	0.4260	0.4240	0.3910	0.3469	0.2882	0.2081
50.00	0.3470	0.3450	0.3131	0.2689	0.2080	0.1310

**Table 7** - Data from numerical calculations of the copper melt motion at 1555 K

x \ y	0.00	-10.00	-20.00	-30.00	-40.00	-50.00
0.00	0.6111	0.6002	0.5721	0.5250	0.4589	0.3752
10.00	0.6002	0.5920	0.5633	0.5159	0.4503	0.3660
20.00	0.5720	0.5630	0.5362	0.4880	0.4223	0.3372
30.00	0.5253	0.5159	0.4882	0.4420	0.3753	0.2903
40.00	0.4590	0.4501	0.4221	0.3750	0.3092	0.2253
50.00	0.3750	0.3660	0.3370	0.2900	0.2250	0.1410

**Table 8** - Data from numerical calculations of the copper melt motion at 1595 K

x \ y	0.00	-10.00	-20.00	-30.00	-40.00	-50.00
0.00	0.6011	0.6432	0.6130	0.5631	0.4922	0.4023
10.00	0.6430	0.6340	0.6028	0.5530	0.4822	0.3922
20.00	0.6132	0.6030	0.5740	0.5230	0.4521	0.3620
30.00	0.5633	0.5532	0.5230	0.4733	0.4020	0.3110
40.00	0.4920	0.4822	0.4520	0.4022	0.3330	0.2412
50.00	0.4021	0.3920	0.3622	0.3110	0.2410	0.1511

**Table 9** - Data from numerical calculations of the copper melt motion at 1635 K

x \ y	0.00	-10.00	-20.00	-30.00	-40.00	-50.00
0.00	0.6988	0.6880	0.6560	0.6020	0.5270	0.4301
10.00	0.6880	0.6772	0.6450	0.5910	0.5160	0.4190
20.00	0.6560	0.6449	0.6131	0.5590	0.4840	0.3871
30.00	0.6020	0.5909	0.5591	0.5060	0.4301	0.3331
40.00	0.5270	0.5160	0.4842	0.4302	0.3562	0.2580
50.00	0.4300	0.4190	0.3870	0.3330	0.2580	0.1620

The physical picture is: when the melt flows through an inclined trough, there is a movement of convective flows or thermodynamic flows, where the bulk force divides the flow into the bottom layer and the main flow. The main flow moves upwards, and the denser lower layer descends, creating circulation and mixing of the medium, as when heated from below. Internal friction forces slow down the movement of the atoms of the lower layer. When the melt flows in an inclined trough, this deceleration is transferred to the surface of the copper melt [8].

The simulation and application of mathematical models for integrated analysis of copper rod production may show manufacturing abnormalities, production efficiency, and reliability.

We have developed a mathematical model for calculating the melt flow rate profile at different temperatures in process equipment, starting from the melting point of copper. We built graphs of isolines of the flow velocities of melted copper in the

technological equipment Copper Rod Production Plant Kazkat. Calculations showed the heterogeneity of the structure of the copper melt near the melting point. Calculations showed that the number of contours at low temperatures, such as 1355 K, is smaller. It was also analytically established that the number of contours at higher temperatures, such as 1595 K, is smaller. This is due to the thermal weakening of the metal properties of the copper melt, which is the cause of structural defects in the production of copper rod.

By analysing early, simulation and application of mathematical models improve decision-making, resource allocation, and operational expenses for process optimization and quality control. The recommended approach to the issues and technological processes for increased efficiency and profitability.

Verification of the melting parameters with manufacturing parameters at Plant Kazkat shows fulfils to optimize the temperature field profile, which leads to an increase in the uniformity of the wire rod from  $0.62 \left[ \frac{m}{s} \right]$  to  $0.05 \left[ \frac{m}{s} \right]$ . Therefore, it is important to take into account that the minimum flow rate of the melt will be at the bottom of the trough, the highest flow rate will be at the surface of the melt. The melting parameters with manufacturing parameters at Plant Kazkat include movement of equations (5) and (6).

Let us calculate the number of isolines at the specified temperatures. The maximum isoline value of the velocity profiles of the copper melt movement at Plant Kazkat equipment at the specified temperatures was 28 at 1555 K, and are presented in Figure 15.

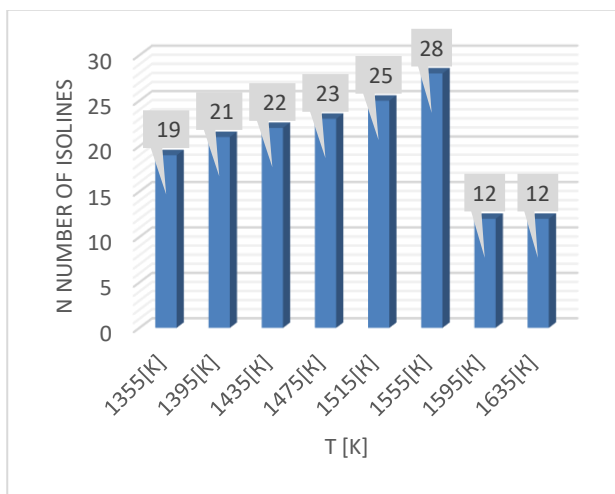


Figure 15 - The isoline value

The numerical methods show that the average value of the mathematical model velocity is

approximately equal to the average velocity of the copper melt flow  $v = 0.4 \left[ \frac{m}{s} \right]$ .

Mathematical and numerical modelling of the copper rod process aims to optimize the temperature field profile. The technological temperature field profile at Plant Kazkat equipment was in the range of 1425-1553 K and shows the correctness and adequacy of our verification of theoretical results.

Below is an example calculation demonstrating the reduction in tensile strength variation and the increase in ductility. We compare 10 batches before and 10 batches after optimization in Table 10.

Table 10 - Batches before and after optimization

Batch No	Tensile Strength $\sigma_u$ , MPa		Similar Example for Ductility $\delta$ , %	
	Before optimization	After optimization	Before optimization	After optimization
1	560	538	21.8	22.6
2	525	542	22.1	22.9
3	545	540	21.5	22.7
4	530	536	22.3	22.8
5	555	544	21.7	22.5
6	520	539	22.0	22.9
7	548	541	21.6	22.6
8	535	537	22.2	22.7
9	562	543	21.4	22.8
10	528	540	21.9	22.6
Mean value	540.8	540.0	21.85	22.71

Calculation of the standard deviation presented at equation (18)

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}} \tag{18}$$

Standard deviation before optimization  $s_1$  and after optimization  $s_2$  is equal to

$$s_1 \approx 14.8 \text{ MPa}, s_2 \approx 2.7 \text{ MPa}.$$

Technological spread  $\pm 2s_1 = \pm 29.6 \text{ MPa}$ .

Technological spread  $\pm 2s_2 = \pm 5.4 \text{ MPa}$ .

So, variance comparison (F-test):

$$F = \frac{s_1^2}{s_2^2}; F = \frac{14.8^2}{2.7^2}; F \approx \frac{219}{7.3} \approx 30.$$

Critical value for n=10 at 95% confidence is 3.18.

Since:

$$F = 30 > 3.1.$$

The difference in variances is statistically significant.

Calculation of ductility increase

$$\frac{22.71 - 21.85}{21.85} - 100\% = 3.9\%$$

Ductility increased by approximately 4%.

As a result of implementing the optimization model, the standard deviation of tensile strength decreased from 14.8 MPa to 2.7 MPa, corresponding to a reduction in technological variation from ±29.6 MPa to ±5.4 MPa. The F-test confirmed statistical significance at a 95% confidence level.

The average elongation increased from 21.85% to 22.71%, corresponding to 39% increase in ductility.

In practice, the following groups of methods are commonly used for wire rod rolling process optimization:

- Empirical technological schedules;
- Regression-based statistical models;
- Finite Element Method (FEM) simulations;
- Machine learning approaches;
- The proposed integrated optimization model.

Advantages of the proposed integrated model combine: physical–mathematical description of deformation; thermal balance modeling; optimization algorithms; statistical validation (Table 11).

Quantitative performance comparison of the proposed integrated model is presented in Table 12.

**Table 11** - Advantages of the proposed integrated model

Criterion	Empirical	Regression	FEM	ML	Integrated Model
Accounts for process physics	–	±	+	–	+
Temperature modeling	–	±	+	±	+
Real-time applicability	+	+	–	+	+
Roll wear prediction	–	–	+	±	+
Adaptability	–	±	–	+	+
Industrial applicability	+	+	±	±	+

**Table 12** - Quantitative performance comparison

Indicator	Empirical	Regression	Integrated Model
Tensile strength variation	±25–30 MPa	±18–20 MPa	±8–12 MPa
Increase in elongation	0–1%	1–2%	3–5%
Roll wear reduction	0–5%	5–8%	15–25%
Scrap reduction	up to 1%	1–2%	2–3%

**Table 13** - Initial assumptions for a medium-capacity plant

Annual production volume	$Q = 120.000t/year$
Average product value	$C_p = 8.500USD/t$
Baseline defect rate	$d_o = 2.5\%$
Expected reduction in defect rate	$\Delta d = 1.0\%$
Annual energy consumption for melting and casting	$E_o = 45.000MWh$
Specific energy consumption, MWh/t	$e_{spec} = 0.52MWh/t$
Expected energy savings	3%
Estimated implementation cost (software integration, training, system calibration)	$I = 450.000USD$
Annual loss due to defects before optimization	$L_o = 25.5millionUSD$
Economic effect after optimization (defect reduction by 1%)	$\Delta L = Q\Delta dC_p = 10.2millionUSD/year$
Annual energy savings:	$E_s = 0.03 \cdot 45.000 = 1.350MWh$
Assuming an electricity cost of	$C_e = 110USD/MWh$
Annual energy savings	$S_e = 1.350 \cdot 110 \approx 148.500USD/year$
Reduction in roll maintenance costs if annual roll maintenance costs	$C_m = 1.2millionUSD/year$
The model reduces wear-related costs by 15%	$S_m = 0.15 \cdot 1.2 \approx 180.000USD/year$
Total annual economic benefit	$S_{total} \approx 10.53millionUSD/year$
Return on investment (ROI)	$ROI = \frac{S_{total} - I}{I} \cdot 100\% \approx 2240\%$
Payback period	$T_{payback} = \frac{I}{S} \cdot 100\% \approx 0.5monts$

Unlike empirical and regression-based approaches, the proposed model provides a comprehensive thermomechanical description of the rolling process and enables multi-objective

optimization. Compared with finite element modeling, the model has significantly lower computational complexity and is suitable for real-time industrial applications. This ensures reduced mechanical property variation, lower roll wear, and improved stability of wire rod quality.

To evaluate the feasibility of implementing the proposed viscosity-based optimization model in copper wire rod production, a quantitative economic analysis was performed.

The implementation of the proposed model demonstrates (Table 13):

- Extremely high return on investment (>2000%);
- Payback period of less than one month;
- Significant reduction in defect-related losses;
- Measurable energy savings;
- Extended roll service life.

Even under more conservative assumptions, a defect reduction of 0.3–0.5%, the payback period remains within several months, confirming strong industrial feasibility.

## Conclusions

A mathematical and computer model and the numerical integration algorithm for calculating the melt flow rate profile at different temperatures in process equipment, starting from the melting point of copper, were developed.

The validation of theoretical results, mathematical model, and the numerical integration algorithm show the recommended approach to the issues and technological processes for increased efficiency and profitability. The proposed model fulfils to optimize the temperature field profile, integration of thermomechanical modeling with an optimization algorithm; introduction of a criterion minimizing mechanical property variation; consideration of reduction and redistribution effects on roll wear; possibility of adaptive real-time control.

The technological temperature field profile at Plant Kazkat equipment was in the range of 1425–1553 K, and the best melting occurred at 1555 K.

The strength of the relationship between the experimental data and the values calculated using it was evaluated using the coefficient of nonlinear

multiple correlation, together with an assessment of its statistical significance. The correlation coefficient  $R=0.952734$  is high, therefore, and shows the correctness and adequacy of our verification of theoretical results.

The numerical calculations show that the average elongation increased from 21.85% to 22.71%, corresponding to 3.9% increase in ductility. Comparison between the proposed model and experimental viscosity data of liquid copper in the temperature range 1355–1635 K demonstrates excellent agreement  $R^2 = 0.91$ , MAPE = 1.34%. The maximum deviation 5.8% occurs near the melting temperature, likely due to structural fluctuations and the assumption of constant activation energy. In the range 1435–1635 K, the model accuracy remains within 1%, confirming its suitability for engineering calculations.

Unlike empirical and regression-based approaches, the proposed model provides a comprehensive thermomechanical description of the rolling process and enables multi-objective optimization, has significantly lower computational complexity, and is suitable for real-time industrial applications.

This ensures reduced mechanical property variation, lower roll wear, and improved stability of wire rod quality. The implementation of the proposed model demonstrates extremely high return on investment (>2000%); payback period of less than one month; a significant reduction in defect-related losses; measurable energy savings; extended roll service life.

**Conflicts of interest.** The authors declare that they have no conflicts of interest in this work.

**CRedit author statement:** **S. Kazhikenova:** Conceptualisation, Methodology, Software; **G. Shaikhova:** Data curation, Writing draft preparation; **S. Shaltakov:** Visualisation, Investigation, Validation; **A. Shaltakova:** Reviewing and Editing.

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## Металлургиядағы технологиялық процестердің тұрақты тиімділігі мәселелерін модельдеу және шешу

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<p>Мақала келді: 20 қаңтар 2026 Сараптамадан өтті: 25 ақпан 2026 Қабылданды: 4 мамыр 2026</p>	<p><b>ТҮЙІНДЕМЕ</b> Мақалада өнім сапасын жақсарту, шығындар мен тәуекелдерді азайту мақсатында мыс илемдік өндірісін интеграциялық талдау үшін модельдеу және математикалық модельдерді құру әдістері қарастырылды. Жүргізілген зерттеу мақсаты нақты процестердің математикалық сипаттамаларын жасау арқылы жүзеге асады, олар кейін компьютерлік тәжірибелер жүргізу үшін қолданылады. Дәстүрлі молекулалық динамикалық әдістерден айырмашылығы, бұл зерттеуде ақпараттық технологияны пайдалану арқылы ағын өрісінің сипаттамаларын технологиялық жабдықтар пайдаланды. Жаңалық келесіде: термомеханикалық модельдеуді оңтайландыру алгоритмімен интеграциялау, критерийлерді енгізу, механикалық қасиеттердің өзгеруін азайту, аударылу кезінде тозуын азайту, нақты уақыт режимінде бейімделгіш басқару мүмкіндігі Kazkat мыс таяқшаларын өндіру зауытында мыс балқыту ағынының кең ауқымдағы жылдамдықтарының профилдерінің ақпараттық талдауы ұсынылған. Ақпараттық технологияларды қолдана отырып, мыс таяқшаларын өндіру процестерінде ең жақсы сапа мен өнімділікке қол жеткізу үшін оңтайлы температура мәндерін, илемдеу жылдамдығын және басқа да параметрлерді анықтау мәселелері шешілді. Осы нәтижелерге сүйене отырып, орамдардағы температура өрісінің профилін оңтайландыру бойынша бағыттар ұсынылды, бұл роликтің тозуын азайтуға және сым таяқшасы құрылымының біркелкілігін арттыруға әкеледі, микроқұрылымды жақсартуға назар аударады.</p>
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## Моделирование и решение задач устойчивой эффективности технологических процессов в металлургии

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<p>Поступила: 20 января 2026 Рецензирование: 25 февраля 2026 Принята в печать: 4 мая 2026</p>	<p><b>АННОТАЦИЯ</b> В статье рассмотрены методы применения, моделирования и создания математических моделей для интегрированного анализа производства медной катанки с целью повышения качества продукции, снижения затрат и минимизации рисков. Цель проводимых исследований достигается путем создания математических описаний реальных процессов, которые затем используются для проведения компьютерных экспериментов. В отличие от традиционных методов молекулярной динамики, применены информационные технологии для получения характеристик поля потока в технологическом оборудовании. Новизна заключается в: интеграции термомеханического моделирования с алгоритмом оптимизации; введение критерия, минимизирующего вариацию механических свойств; рассмотрение влияния перераспределения уменьшения на износ переворота; Возможность адаптивного управления в реальном времени. Представлен информационный анализ профилей скоростей течения расплава меди в широком диапазоне температур на заводе</p>
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	Казкат по производству медной катанки. С помощью информационных технологий решены задачи определения оптимальных значений температуры, скорости проката и других параметров для достижения наилучшего качества и производительности процессов производства медной катанки. На основании проведенных исследований предложены направления оптимизации профиля температурного поля в валках, что приводит к снижению износа рулона и увеличению равномерности структуры катанки, с акцентом на улучшение микроструктуры.
	<b>Ключевые слова:</b> Моделирование, медь, компьютерное моделирование, расплав, информационные технологии, численный метод.
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