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Comparative Analysis of Mathematical Models of Drilling in Heterogeneous Geological Sections

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ABSTRACT
This paper presents a comparative analysis of four main types of well drilling models in heterogeneous geological sections: mechanical-mathematical, energy, kinematic, and empirical. It is shown that each group of models focuses on different aspects of the process: the physics of bit-rock interaction (mechanical-mathematical approach), the energy efficiency of rock mass destruction (energy), the trajectory and movement of the tool (kinematic), as well as statistical patterns and the prediction of complications (empirical). The interaction between the bit and the rock is considered depending on their physical and mechanical properties. A comparison of the rotation speed of the rotor and bit is provided depending on the rock hardness. Based on a review of modern publications and the practical experience of leading service companies (Equinor, Schlumberger, Halliburton), the strengths and weaknesses of each approach are identified, and the need for their integration is substantiated. It is established that the integrated use of models of different classes allows not only to describe and explain phenomena but also to manage the drilling process in conditions of high geological variability.

Keywords: drilling, heterogeneous formations, mechanical-mathematical models, energy models, kinematic models, empirical models, stick–slip, digital drilling.

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Indroduction

Drilling in heterogeneous formations is always characterized by high complexity and uncertainty, due to the variability of lithological properties of rocks, the presence of tectonic fault zones, fracturing, and variations in the physical and mechanical properties of the rock mass [1], such as soft layers alternating with hard ones, cracks and caverns, and the entire system reacts to this with speed surges, circulation losses, and premature bit wear [2]. One of the most severe situations remains the absorption of drilling fluid in fractured and cavernous zones—a process that halts work and requires immediate costs to eliminate. Unless such intervals are predicted in advance, shutdowns are inevitable. Therefore, today there are more than a hundred standard sizes and modifications of drilling tools worldwide. The search for new, more effective modifications of drill bits continues, since rock destruction during drilling is an unsteady process [3].

Modern well drilling is characterized by heterogeneous geological sections and uncertainty. Under these conditions, classical engineering approaches based on a limited set of parameters prove insufficient to accurately predict the behavior of the rock-drilling tool-wellbore system.

To analyze rock fracture processes and drilling dynamics, various classes of models are used in scientific literature and industrial practice: mechanical-mathematical, energy, kinematic, and empirical [[4],[5]]. Each of these models reflects individual aspects of a complex process. Thus, mechanical-mathematical models focus on the physics of bit-rock interaction [6], energy models on assessing fracture efficiency and specific energy costs [7], kinematic models on the dynamics of drill string motion and vibration processes, and empirical models on statistical dependencies and the prediction of complications using big data and machine learning methods [8].

In recent years, there has been a trend toward integrating these approaches: leading service companies such as Equinor, Schlumberger, and Halliburton demonstrate successful examples of combining physical models with data-driven analytics, which improves the reliability of forecasts, improves the controllability of the drilling process, and reduces technical and economic risks [[9],[10]]. Thus, the relevance of the study is determined by the need to systematize existing models, identify their strengths and weaknesses, and substantiate the viability of an integrated approach that ensures reliable and effective management of the drilling process in conditions of high geological variability.

Heterogeneity is also manifested in the destruction of the massif. For example, in drilling and blasting operations, the Kuz–Ram model is widely used (an empirical relationship for predicting the granulometric composition of broken rock), but in real heterogeneous massifs, its results turn out to be idealized: digital processing of images of dumps shows a strong discrepancy between the calculated and actual distribution of fragment sizes. In other words, a simple model does not hold up when confronted with complex geology, where fracturing and differences in strength and structure create an unpredictable failure pattern.

Therefore, the question becomes particularly important: which models can adequately reflect environmental variability? Today, there are four directions. Mechanical-mathematical models are based on the laws of mechanics and allow one to see the physical nature of the process, but require accurate data on the strength of rocks and are difficult for direct use at the drilling site. Energy models reduce the process to integral indicators - specific energy of destruction, mechanical penetration rate - and are convenient for diagnosing efficiency, although they remain blind to the details of the interaction of the bit with the rock. Kinematic models consider the movement of the tool and the trajectory of the well, are visual and applicable in navigation, but describe the process only geometrically. Empirical approaches, including modern neural network algorithms, can quickly predict complications and drilling parameters, but their reliability is limited by the training set.

This is how the field of comparison is built: mechanical and mathematical ones give a fundamental picture, energy ones - economics and efficiency, kinematic ones - the geometry of motion, empirical ones - a statistical forecast. The question is which of these approaches copes better with the description of drilling specifically in the conditions of lithological mosaic and variable rock strength - the accuracy of the forecast of the rate of penetration, the assessment of energy costs, and the stability of the tool depend on this.

Methodology

Modern drilling research in heterogeneous formations relies on diverse methodological foundations, reflecting the complexity of the problem itself. University and academic centers predominantly focus on fundamental approaches—numerical modeling based on the equations of solid mechanics and the dynamics of multiphase systems. Here, the emphasis is on constructing integrated "drilling rig–drill string–bit–rock" models, incorporating nonlinear friction, contact interactions, and three-dimensional stress distribution. Such studies are verified through bench tests or limited borehole data, and they form the basis for understanding phenomena such as stick-slip or wellbore stability [11].

Industrial companies operate differently: they focus on energy indicators and fast heuristic algorithms. Schlumberger, Halliburton, and a number of Russian service companies are integrating mechanical specific energy (MSE) and energy efficiency coefficient calculations into their monitoring systems, combining them with data streams from WOB (weight on bit), RPM (revolutions per minute), and circulating system pressure sensors. The methodology is pragmatic: models must operate in real time, so physical detailing gives way to adaptive calibration algorithms.

A separate area of research is kinematic and structural modeling, which is being actively developed by geophysical divisions. 3D seismic data, attribute analysis of wells, and integration with GIS (well logging) are used to construct structural-kinematic models of deposits. These models do not directly describe the rock failure process, but rather define the geological framework within which mechanical and energy algorithms operate.

Finally, the most rapidly developing empirical methodology is machine learning and intelligent forecasting systems. Researchers from China, the USA, and Russia are training neural networks and hybrid models on large arrays of drilling data, including telemetry, lithological sections, and emergency events. Here, data preparation becomes key: automatic filtering, channel synchronization, and the correct marking of complications. It is at this stage that new methodological developments are concentrated, from signal preprocessing systems to the concept of a "digital twin" of a well. Thus, the research methodology has broadly evolved into four areas: fundamental numerical modeling, applied energy indicators, geophysical kinematics, and empirical big data analytics. In our work, we build on these approaches, comparing their strengths and weaknesses and analyzing the potential for integration in heterogeneous geological settings.

Results

It is advisable to begin a systems analysis with mechanical and mathematical models, as they reflect the fundamental physics of drilling and provide the basis for more applied energy and empirical approaches. Thus, A.N.

Petrov and S.V. Gavrilyuk demonstrate that a model in which the drilling rig, string, bit, and rock are considered as a single system with multi-mass dynamics, contact friction at the bottomhole, and electromechanical drive allows for the reproduction of key self-oscillatory modes [12]. Numerical experiments confirmed that, under certain parameters, a stick-slip effect occurs—an alternation of stops and jerks of the bit with a drop in average angular velocity and an increase in parasitic energy consumption (see Fig. 1). These findings echo the results of M.A. Sidorov and I.L. Kuznetsov, who demonstrated that torsional vibrations cause impact loads, accelerate wear on PDC bits, and increase the risk of threaded connections loosening (see Fig. 2) [13]. Similar observations are made by V.V. Isaev and D.S. Rogov, emphasizing that it is precisely these modes that "eat up" the mechanical penetration rate and tool life.

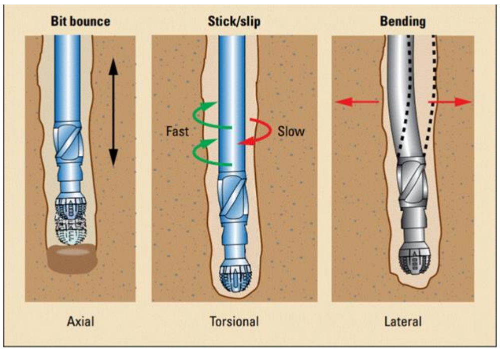

Figure 1 - Types of drill string vibrations: axial (bit bounce), torsional (stick–slip), and lateral (bending)

A.V. Barkov's geomechanical research line expands our understanding of the process: without taking into account the block and fracture structure of rocks, wellbore stability predictions are inevitably distorted. Calibrating numerical models using triaxial core testing and computed tomography of fractures allows for significantly more accurate predictions of the stress-strain state of the rock mass and potential complication zones. Thus, the mechanical-mathematical layer explains the causes of ROP degradation and the occurrence of emergency conditions, but due to its high demands on data and computing resources, its application in online practice is limited.

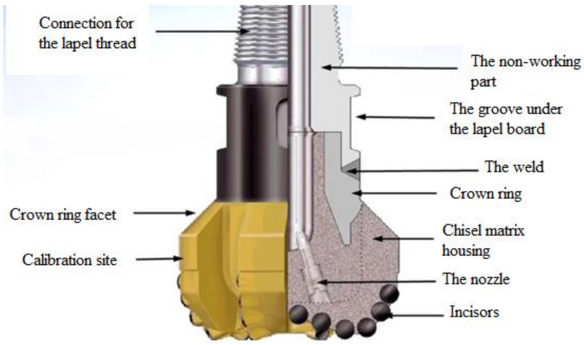

Figure 2 - Diagram of a PDC bit in section showing the main structural elements that determine the dynamics of interaction with the rock

Against this backdrop, energy models have become a practical tool, as they reduce the complex process to integral indicators of "expended power - destruction volume." The central criterion here is the mechanical specific energy, first proposed by R. Theale back in 1965, as the amount of energy required to destroy a unit volume of rock. The meaning of this indicator is simple: the lower the MSE value, the more efficient the drilling. In practice, MSE is calculated based on telemetry data: weight on bit (WOB), rotational speed (RPM), torque, and rate of penetration. The classical model of A.T. Bourgoine and F.S. Young, based on regression, made it possible to define the "WOB-RPM" working areas and delineate zones of uniform rotation, areas of instability with stick-slip characteristics, and zones of lateral oscillations, where efficiency drops sharply (see Fig. 3) [13].

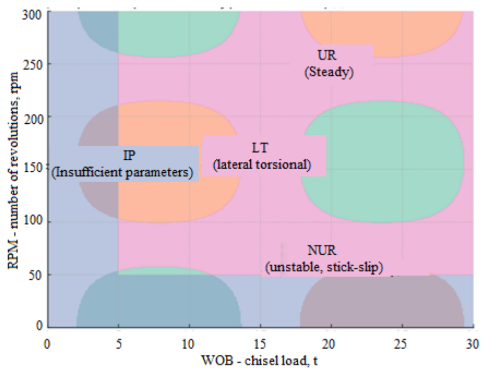

Figure 3 - Drilling mode diagram in WOB–RPM coordinates, demonstrating the boundaries of stable bit rotation (UR), the region of unstable mode with manifestations of stick–slip (NUR), the zone of insufficient parameters (IP), and the region of lateral torsional oscillations of the column (LT)

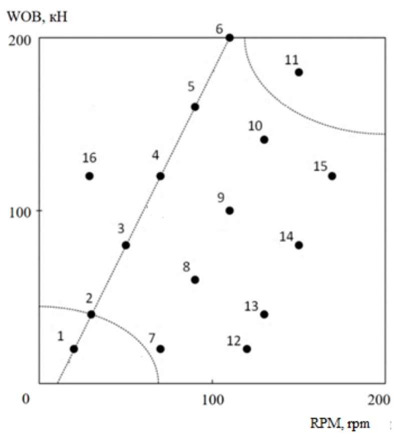

Figure 4 - Rotary drilling mode diagram in WOB–RPM coordinates: stable rotation zone (UR), unstable mode zone with stick–slip manifestations (NUR), insufficient parameters zone (IP) and lateral column vibration zone (LT)

The applicability of the energy approach is confirmed by the experience of service companies Schlumberger and Halliburton: in the field, MSE spikes are used to determine whether the bit is entering a harder formation, while dips indicate entry into loose or fractured zones with a risk of circulation loss [[14],[15]]. This has become a standard technique in operational geosteering, as evidenced by examples from the Permian Basin and the North Sea, where energy indicators helped quickly localize ineffective performance windows and return the process to the optimal parameter range.

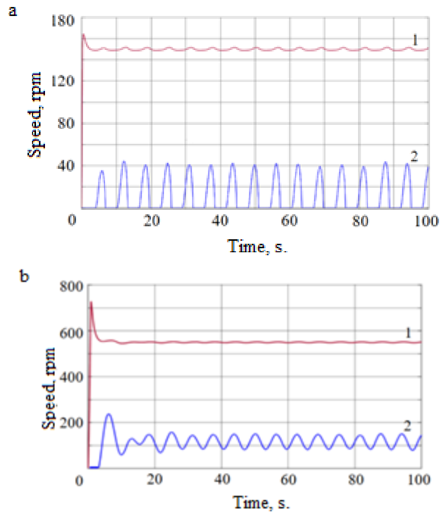

Figure 5 - Comparison of the rotation speed of the rotor and bit: mode 16 - unstable rotation (stick-slip), mode 9 - uniform rotation with a high mechanical penetration rate [16]

The transition from purely diagnostic criteria to automation has become a key step in the development of energy and kinematic models. Drilling mode diagrams in WOB–RPM coordinates clearly show the limits of stable rotation, instability zones with stick-slip behavior, as well as areas of lateral oscillation and insufficient parameters (Fig. 4). These zones serve as the basis for digital monitoring systems, allowing for real-time detection of critical conditions. Against this backdrop, companies began implementing closed-loop control systems: Equinor's experience with downhole regulators and automatic control demonstrated that suppressing torsional oscillations and aligning rotation leads to increased ROP and reduced performance variability; in pilot projects at Schlumberger-Equinor's Brazilian projects, the rate of penetration increase reached 60%. A detailed comparison of rotation dynamics in various modes reveals the nature of these effects: with uniform rotation, energy is transferred to the face without loss, whereas in stick-slip mode, alternating stops and jerks, a decrease in average angular velocity, and torque surges are observed (Fig. 5). This instability inevitably increases energy consumption and accelerates wear of PDC bits. Therefore, energy optimization requires consideration of the actual dynamics of the process: the fractional-energy model of A. A. Pavlov and S. N. Grigoriev demonstrates that rationalizing WOB and RPM simultaneously reduces energy consumption, increases the average cutting size, and extends the service life of the tool [17].

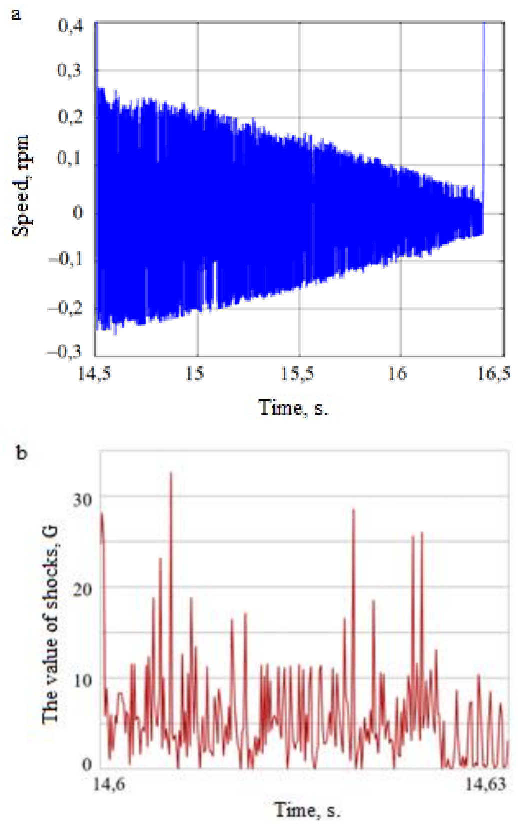

Figure 6 - High-frequency vibrations of the bit in stick-slip mode, accompanied by impact loads and accelerated wear of the cutting elements [17].

High-frequency vibrations of the bit, occurring in advanced stick-slip mode (Fig. 6), pose a particular danger. They are accompanied by impact loads, causing accelerated wear of cutting elements and creating the risk of threaded connections becoming loose. In practice, this is one of the most dangerous scenarios, which is why modern drilling control systems are focused on preventing entry into such zones rather than mitigating the consequences. Taken together, this confirms that only the integration of mechanical-mathematical models, energy indicators, and intelligent control algorithms allows the process to be maintained within a safe and efficient parameter range.

The kinematic modeling level defines the geometric basis of the drilling process, where the object of analysis is not the force interaction, but the trajectory of the system elements and the wellbore. In engineering practice, this manifests itself in two dimensions: first, as the kinematics of the drill string and tool, and second, as the kinematics of the geological structures that determine the borehole curvature. In the first case, kinematic diagrams of the drive, column, and bit are constructed to optimize the transmission of rotation and axial load to the face [18]. When considering heterogeneous sections, the actual kinematics of the deepening process become more important: a geometric estimate of the maximum possible rate of penetration (ROP), which sets the upper limit of efficiency. Comparing the actual ROP with the "ideal" ROP shows the extent to which the bit's movement is limited by the strength properties of the rock. This principle is embedded, in particular, in the Burgoyne-Young model, where the kinematic component of the velocity is adjusted by empirical coefficients reflecting the influence of lithology and hydraulics.

The second line is the wellbore trajectory as it passes through zones with varying drillability and structural complexity. Faults and inclined layers can cause bit deviation, so structural-kinematic models are used to predict curvature and select the optimal trajectory. For example, T.Sh. Akhmedov and K.N. Ibragimov, using the integration of 3D seismic with well logging, constructed a model of the Kurovdag field, revealing hidden tectonic blocks and faults not visible in the original sections. The resulting model made it possible to refine the course of horizontal wells and avoid complications when crossing tectonic zones [19].

Real-time kinematic analysis is performed using MWD and LWD systems, which record azimuth, inclination, and changes in logging parameters. The use of azimuthal methods, as demonstrated by A.R. Isaev and all using density logging data while drilling to identify the boundaries of hydrodynamic isolation between reservoirs and adjust the subsurface model. Essentially, the kinematic approach transforms drilling into a tool for "sounding" the subsurface, where the trajectory and its changes themselves become diagnostic indicators of geological heterogeneity.

The oscillatory modes of the drill string deserve special mention. Although they are described dynamically, in practice, only kinematic parameters—vibration amplitudes and frequencies—are often recorded. This allows for the identification of critical modes, such as backward whirl, when the string strikes the borehole walls. A geometric model of the bit center's motion relative to the wall allows for the determination of critical frequencies and their comparison with the LT zones on the WOB–RPM diagrams. Thus, even without full dynamics, kinematic analysis helps identify the conditions that lead to hazardous modes.

A modern trend is the automation of trajectory and mode control. An example is the Downhole Regulators tested by Equinor. This device, mounted on the bit, adjusts the feed rate in real time, smoothing out oscillations and preventing stalls. Reported data from 93 well sections demonstrate an increase in ROP and a reduction in the number of complications, confirming the effectiveness of using an on-the-fly kinematic model as part of an intelligent drilling autopilot.

Collectively, kinematic models form a tool for operational diagnostics and geosteering: they enable the identification of hidden heterogeneities, trajectory prediction, and operational adjustments before complications arise. Their limitation remains their inability to determine the causes of these phenomena: they record movement but do not explain its physics. Therefore, their maximum effectiveness is achieved when combined with mechanical-mathematical and energy models that reveal the hidden force mechanisms underlying the observed kinematics.

Discussion

The approaches to drilling modeling discussed above-mechanical-mathematical, energy, kinematic, and empirical-are not mutually exclusive. On the contrary, they complement each other, each providing its own perspective on the complex, multifactorial process of rock failure. In real-world heterogeneity, effective drilling planning and execution require a combination of all four model types.

Mechanical-mathematical models provide physical plausibility and the ability to extrapolate system behavior beyond existing experience. They allow us to explore new drilling modes, materials, and bit designs. For example, they can be used to predict the occurrence of stick-slip and other vibrations at specific WOB and RPM ratios, which has been confirmed in practice. However, the complexity of such models hinders their application directly at the drilling rig: computational power and time are required to solve the equations, and, most importantly, input data is not always available (for more details, the rock parameters for each meter of the borehole are unknown in advance). Therefore, in real time, it is difficult to rely solely on a purely physical model.

Energy and kinematic models fill this gap, enabling rapid process control. They are simple and quick to calculate. Specific energy mode diagnostics, as noted, instantly demonstrate drilling efficiency; trajectory and speed monitoring detect deviations when entering a new drilling phase. However, if an abnormal situation arises, these models only record the symptom (increased MSE, trajectory drift) but do not explain the cause. Engineering expertise, a connection to a physical (mechanical) model, or big data analysis should be used here [[20],[21]].

Empirical (data-driven) models allow us to account for the complexities of geology without delving into the physics of the processes. They work well as a warning and prediction system: for example, a neural network can predict that lost circulation is likely within 5 meters, based on a combination of indirect indicators, even before it manifests itself. However, such predictions are always subject to error, and they offer no guarantee beyond known patterns. This is why the best strategy is to integrate models. Modern automated drilling systems (so-called intelligent drilling rigs) are built on a three-tiered structure: at the bottom tier are sensors and direct automation (kinematics and energy: maintaining a set flow rate, pressure, and power); at the middle tier are optimization models and algorithms (physical and empirical models that evaluate the process state); at the top tier is an operator or AI that makes decisions based on the combined information. All of the described approaches fit naturally into this architecture.

Table 1 - Comparative characteristics of drilling models in heterogeneous sections.

Model approach	Main features and advantages	Limitations and disadvantages
Mechanics and Mathematics	Accurate physical description of the process; prediction of new effects; consideration of material properties	Complexity of implementation; requires many input parameters; high computational costs; difficult to apply in real time
Energy	Integrated performance assessment (specific energy, efficiency); rapid diagnostics of deteriorating conditions; optimization of modes to minimize energy consumption	Does not show the causes of the phenomenon; may misinterpret complex cases (for example, an increase in MSE due to vibrations or hard rock); requires calibration for conditions
Kinematic	Drilling trajectory and speed control; fault and rock change detection based on course/ROP changes; simple motion models for automation	Does not take into account forces and resistances; without a connection to dynamics, it can give a simplified representation; it does not prevent problems, but only records movement
Empirical (data)	Considers multiple factors simultaneously; self-learning based on accumulated data; forecasts complications and parameters in real time; adapts to a specific field	Requires large, high-quality training data; is limited to a domain similar to the training sample; is a "black box"—provides a prediction without explaining the physics of the process

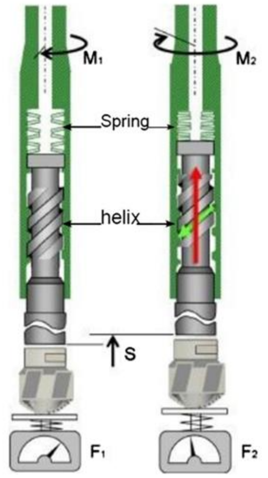

Figure 7 - Operating principle of the AST, which smooths out torsional vibrations and stabilizes the rotation of the bit when drilling in heterogeneous sections [22].

Equinor's experience with the implementation of downhole weight-on-bit (AST) regulators exemplifies successful integration. The device is based on an understanding of stick-slip mechanics: a physical and mathematical model demonstrated that a spring-damper element integrated into the string can smooth out intermittent torsional vibrations and stabilize rotation. Kinematic analysis allowed us to define the deformation parameters and speed range at which the regulator operates effectively; energy calculations confirmed its impact on reducing the specific energy of fracture and increasing ROP, and empirical statistics from dozens of wells demonstrated the reliability of the solution. The diagram (Fig. 7) illustrates the operating principle of the AST: as torque increases, the tool compresses elastically, reducing the weight on the bit and preventing instability. According to reports, the implementation of such regulators has resulted in a 30–40% increase in ROP and a significant reduction in accidents, despite the complex structure of the wellbore. Another example is digital twins and drilling simulators, which are currently being developed in research centers. They combine a geological model of the section (usually stochastic, accounting for heterogeneity), a geomechanical model (stress, wellbore stability), a bit and string model (mechanics and kinematics), and an integrated ML model calibrated against actual well data. Such a twin can compare forecasts and actual performance online and adjust either model parameters or recommend a change in drilling mode [[23],[24]]. Essentially, this is the fusion of all four approaches into a single software suite.

As Table 1 shows, no single model type covers all aspects of a complex drilling problem. Therefore, in engineering practice, combinations are used. For example, when planning a bit and drilling mode for a new section, mechanical and mathematical calculations are first used (to estimate the required force, torque, and vibration risk). Then, during drilling, energy monitoring (MSE and power monitoring) and kinematic control (navigation trajectory and mode stabilization using vibration sensors) are used. Empirical trends are also constantly compared (models trained on adjacent wells suggest expected values and warn of abnormal ones) [25]. This integrated approach is the most reliable.

This is especially important for heterogeneous sections, where the situation can change abruptly. Here, a mechanical model will warn in advance: "stick-slip is possible in interval X with the given parameters" – this allows for reconfiguring the drilling mode before entering the interval. When drilling enters an interval, energy indicators will show the actual state: "energy consumption is increasing, efficiency is decreasing" – a signal to review the WOB/RPM. Kinematics and sensors will indicate: "the bit has started to deviate/vibrate" – perhaps a fault or lens has been encountered, requiring a reduction in speed or the use of dampers. An empirical system can also provide the following: "Based on experience, in such cases, there will be a loss of power after 10 meters, so be prepared." Using this information together significantly increases the chances of successfully drilling through a challenging interval.

Interest in the automation and intellectualization of the drilling process has grown significantly in recent years. By 2025, drilling rigs where humans merely control the actions of an automated system that optimizes drilling according to a predefined algorithm will already be a reality. However, models cannot be completely eliminated—they constitute the "brain" of such a system. A promising area of research is the development of adaptive models that update their parameters during drilling. For example, a mechanical model can adjust rock strength properties based on actual load and speed data, i.e., adapt to the actual cross-section rather than the nominal one. Empirical models, in turn, can integrate physical constraints (for example, learning not from meaningless correlations, but by taking into account known physical relationships between parameters). All this is aimed at making drilling in highly heterogeneous conditions more predictable and controllable.

Conclusion

The review showed that drilling in geologically heterogeneous sections can only be described using a comprehensive approach: individual models capture important aspects, but only their combination can explain and predict the actual process dynamics. Based on mechanical, mathematical, energy, kinematic, and empirical studies, the following conclusions were reached:

1. Mechanico-mathematical models explain key phenomena: numerical models of the string and bit (A. N. Petrov, S. V. Gavrilyuk, M. A. Sidorov, I. L. Kuznetsov) reproduce stick-slip and torsional self-oscillations, demonstrating how they reduce ROP and destroy the PDC tool; geomechanical calculations (A. V. Barkov) demonstrated that without taking fracturing and block structure into account, well stability predictions are distorted.
2. Energy models capture the process economics: MSE calculations and WOB–RPM diagrams localize inefficiencies, and the fractional-energy model (A. A. Pavlov, S. N. Grigoriev) confirmed that optimization of the drilling regime reduces specific costs and simultaneously increases bit life. In tests, this resulted in reduced energy consumption and an increase in the yield of coarse cuttings.
3. Kinematic models have practical applications in geosteering and trajectory control: the work of T.Sh. Akhmedov and K.N. Ibragimov demonstrated that attribute analysis of seismic and well logging allows for the prediction of block displacements and faults; the methodology of O.I. Ilmendeeva and V.A. Nosov demonstrated that LWD logging detects hydrodynamic barriers during drilling, allowing for rapid course corrections.
4. Empirical and predictive models have demonstrated effectiveness in predicting complications: neural network developments (A.S. Kiselev, S.V. Nechaev, E.G. Gurinov, N.Yu. Klyuchnikov) successfully predicted lost circulation and stuck wells several meters before they occurred, demonstrating the value of machine learning in complex cross-sections.
5. Industrial practice (Equinor, Schlumberger, Halliburton) has confirmed that the integration of approaches is already producing results: the implementation of AST has reduced the amplitude of oscillations, stabilized rotation, and ensured an increase in ROP by 30–40%.

Conflict of interest. On behalf of all the authors, the corresponding author declares that there is no conflict of interest.

CRediT author statement**: J. Toshov, M. Rabatuly:** Conceptualization, Methodology, Software; **D. Erkinov, B. Baratov**: Data curation, Writing-Original draft preparation; **A. Yesendosova, N. Malybaev:** Visualization, Investigation; **D. Erkinov, J. Toshov:** Software, Validation.

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Біртекті емес геологиялық учаскелерде бұрғылаудың математикалық модельдерін салыстырмалы талдау

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ТҮЙІНДЕМЕ
Бұл жұмыста біртекті емес геологиялық учаскелердегі ұңғымаларды бұрғылау модельдерінің төрт негізгі түріне салыстырмалы талдау ұсынылған: механикалық-математикалық, энергетикалық, кинематикалық және эмпирикалық. Әрбір модель тобы процестің әртүрлі аспектілеріне бағытталғаны көрсетілген: қашаудың тау жынысымен өзара әрекеттесу физикасы (механикалық-математикалық тәсіл), тау жынысы массасының бұзылуының энергия тиімділігі (энергия), құралдың траекториясы мен қозғалысы (кинематика), сондай-ақ статистикалық заңдылықтар және асқынуларды болжау (эмпирикалық). Қашау мен жыныстың өзара әрекеттесуі олардың физикалық және механикалық қасиеттеріне байланысты қарастырылады. Ротор мен қашаудың айналу жылдамдығы тастың қаттылығына байланысты салыстырылады. Заманауи басылымдарға шолу және жетекші қызмет көрсету компанияларының (Equinor, Schlumberger, Halliburton) практикалық тәжірибесі негізінде әрбір тәсілдің күшті және әлсіз жақтары анықталып, оларды біріктіру қажеттілігі негізделеді. Әртүрлі кластағы модельдерді кешенді пайдалану тек құбылыстарды сипаттауға және түсіндіруге ғана емес, сонымен қатар жоғары геологиялық өзгергіштік жағдайында бұрғылау процесін басқаруға мүмкіндік беретіні анықталды.

Түйін сөздер: бұрғылау, біртекті емес қабаттар, механикалық-математикалық модельдер, энергетикалық модельдер, кинематикалық модельдер, эмпирикалық модельдер, қолмен сырғыту, сандық бұрғылау.

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Сравнительный анализ математических моделей бурения скважин в неоднородных геологических разрезах

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АННОТАЦИЯ
В данной статье представлен сравнительный анализ четырех основных типов моделей бурения скважин в неоднородных геологических разрезах: механико-математических, энергетических, кинематических и эмпирических. Показано, что каждая группа моделей фокусируется на различных аспектах процесса: физике взаимодействия долота с горной породой (механико-математический подход), энергетической эффективности разрушения массива горной породы (энергия), траектории и движении инструмента (кинематика), а также статистических закономерностях и прогнозировании осложнений (эмпирический). Взаимодействие между долотом и породой рассматривается в зависимости от их физических и механических свойств. Приводится сравнение скорости вращения ротора и долота в зависимости от твердости породы. На основе обзора современных публикаций и практического опыта ведущих сервисных компаний (Equinor, Schlumberger, Halliburton) выявлены сильные и слабые стороны каждого подхода и обоснована необходимость их интеграции. Установлено, что комплексное использование моделей разных классов позволяет не только описывать и объяснять явления, но и управлять процессом бурения в условиях высокой геологической изменчивости.

Ключевые слова: бурение, неоднородные пласты, механико-математические модели, энергетические модели, кинематические модели, эмпирические модели, ручное скольжение, цифровое бурение.

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