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Structure of turbulent non-isothermal flow in a pipe with a sudden expansion

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ABSTRACT
The article studies a mathematical model of turbulent non-isothermal flow of non-Newtonian fluid. At the inlet, the fluid is Newtonian and, due to a decrease in temperature, it becomes non-Newtonian due to increased viscosity and yield strength. The system of turbulent motion and heat transfer equations is solved by the numerical control volume method in variables of the velocity and pressure components. The calculations yielded average and pulsation characteristics of the non-isothermal motion of non-Newtonian fluid in a pipe with sudden expansion. The calculations show a sharp reduction in the structure of the recirculation zone and a decrease in its parameters with an increase in the Bingham number Bn. In this zone, the maximum negative value of the average velocity, equal to–Umax/Um1 ≈ 0.2 for a Newtonian fluid, decreases to –Umax/Um1 ≈ 0.1 at the Bingham number Bn = 17. A decrease in the turbulent characteristics of the non-Newtonian fluid flow is also observed with an increase in the Bingham number. Heat exchange characteristics in the flow region of turbulent non-Newtonian and Newtonian fluids are qualitatively similar. The location of the flow attachment and maximum heat exchange of non-Newtonian fluid does not exceed 10%. The length of the recirculation zone of viscoplastic fluid is shorter by up to 66% compared to Newtonian fluid.

Keywords: non-isothermal turbulent flow, viscoplastic fluid, recirculation zone of pipe flow with sudden expansion.

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Introduction

Turbulent fluid flow with sudden expansion in pipes is used to intensify heat exchange in various technical devices. Such a design of pipe connections takes place in centralized heating systems. This primitive geometry was used at a fundamental level to study the flow division pattern. With the sudden expansion of the flow in the pipe, the static pressure increases and the kinetic energy of the fluid decreases, a recirculation zone is formed, which divides the flow into two parts. Behind the recirculation zone, the flow fills the entire cross-section of the pipe with a flat velocity profile. The separation surface becomes unstable at moderate and high Reynolds numbers (Re) and is the source of vortex generation in both parts of the flow. Moreover, the vortices develop and gradually disappear. This is because the liquid flows against the increase in static pressure with the formation of a recirculation zone. The flow after a sudden expansion of the pipe cross-section can be considered as a jet in an annular recirculation region. This jet expands radially in the expansion region of the pipe until it reattaches to the pipe wall. As experimental studies of fully developed turbulent flow in pipes show [[1], [2]], reattachment of the flow occurs at lengths 6-9 times greater than the height of the expansion stage of the pipe.

In [3] the separation and reattachment of a turbulent flow in a pipe with sudden expansion were also studied experimentally. Experimental measurements of the velocity field were carried out by several authors [[4], [5], [6]] at moderate and high Reynolds numbers of the flow in a pipe with sudden expansion. These measurements showed that the separation region contains large velocity gradients and high shear in combination with an unfavourable pressure gradient. Whereas the reverse flow velocities in the recirculation region are of the order of 10% of the average velocity in the expanding part. The presence of the pipe wall prevents fluid entrainment so that an unstable vortex structure is established further downstream. As experiments [[4], [5], [6]] show, it is difficult to obtain accurate quantitative measurements of turbulence in the recirculation zone, since the average velocities are usually small and the turbulence intensity is high. In this paper, the calculated data of a turbulent non-isothermal flow of a non-Newtonian fluid in a pipe with sudden expansion are presented.

Statement of the problem. Figure 1 shows the flow diagram.

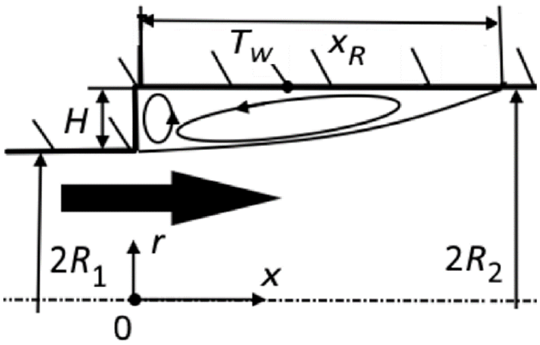


Figure 1 - Flow diagram in a pipe with a sudden expansion

A Newtonian fluid with a temperature T_1 = 303 K and an average velocity u_1 flows into a pipe. The ambient temperature T_w = 273 K is less than the temperature of the incoming fluid. Heat exchange with the environment leads to cooling and a decrease in the temperature of the fluid. A decrease in temperature increases the viscosity and yield point, causing the transition of the Newtonian fluid to non-Newtonian. It is required to find the patterns of average and pulsation characteristics of turbulent non-isothermal flow of a non-Newtonian fluid in a pipe with a sharp expansion.

The system of equations of non-isothermal turbulent motion and heat transfer of a viscoplastic fluid is written in the form [[7], [8], [9]]:

$$\nabla \cdot \mathbf{U} = 0 \qquad (1)$$

$$\nabla \cdot (\rho \mathbf{U}\mathbf{U}) = -\nabla P + \nabla \cdot (2\mu_{eff}\mathbf{S}) + \nabla \cdot \left(-\rho\langle \mathbf{u}'\mathbf{u}'\rangle\right) + \nabla \cdot \langle 2\mu_{eff}'\mathbf{S}'\rangle$$

(2)

$$\nabla \cdot \left(\rho C_p T \mathbf{U}\right) = \nabla \cdot (\lambda \nabla T) + \nabla \cdot \left(-\rho C_p \langle \mathbf{u}'t'\rangle\right) + \boldsymbol{\tau} : \nabla \mathbf{U}$$

(3)

The coefficient of effective molecular viscosity μ_{eff} of a viscoplastic liquid is found from the expression [[10], [11], [12], [13], [14]]:

$$\mu_{eff} = \begin{cases} \mu_p + \tau_0 |\dot{\gamma}|^{-1}, & |\tau| \quad \tau_0 \\ \infty, & |\tau| \le \tau_0 \end{cases} \qquad (4)$$

The singular property $|\tau| \le \tau_0$ of formula (4) can be regularized using the approach [15] and written as:

$$\mu_{eff} = \mu_p + \frac{{}_0\left[1 - \exp\left(-m|\dot{\gamma}|\right)\right]}{|\dot{\gamma}|} \qquad (5)$$

Where the regularization parameter is m=1000 s [15].

The turbulent stress model (RSM model) [[9], [16]] is used to close the system of equations of motion and heat transfer (1-5):

$$\frac{\partial}{\partial x_j}\left(\rho U_j \langle u_i' u_j' \rangle\right) = \rho\left(P_{ij} + \phi_{ij} - \varepsilon_{ij}\right) + \frac{\partial}{\partial x_l}\left[\rho \nu_{eff}\delta_{lm} + \rho \frac{C_\mu T_T}{\sigma_k}\langle u_l' u_m' \rangle\right]\frac{\partial}{\partial x_m}\langle u_i' u_j' \rangle + D_{NNF}$$

$$\frac{\partial}{\partial x_j}\left(\rho U_j \varepsilon\right) = \frac{1}{T_T}\left(C_\varepsilon \tilde{P} - C_{\varepsilon 2}\varepsilon\right) + \frac{\partial}{\partial x_l}\left[\rho \nu_{eff}\delta_{lm} + \rho \frac{C_\mu T_T}{\sigma_\varepsilon}\langle u_l' u_m' \rangle\right]\frac{\partial \varepsilon}{\partial x_m} + \varepsilon_{NNF}$$

$$\chi - L_T^2 \nabla^2 \chi = 1 . \qquad (6)$$

In [[9], [16]] the constants and closing functions of the system of equations (6) are given.

Boundary conditions. The flow diagram is shown in Figure 1.

On the pipe wall before and after expansion:

$$U = V = \langle \mathbf{u}'\mathbf{u}' \rangle = 0 \text{; } T = T_W = \text{const;}$$

$$\varepsilon = 2\nu_W \frac{k}{y^2} \text{; } \chi = 0 \qquad (7)$$

On the pipe axis:

$$\frac{\partial U}{\partial r} = V = \frac{\partial T}{\partial r} = \frac{\partial \langle \mathbf{u}'\mathbf{u}' \rangle}{\partial r} = \frac{\partial \varepsilon}{\partial r} = \frac{\partial \chi}{\partial r} = 0 \qquad (8)$$

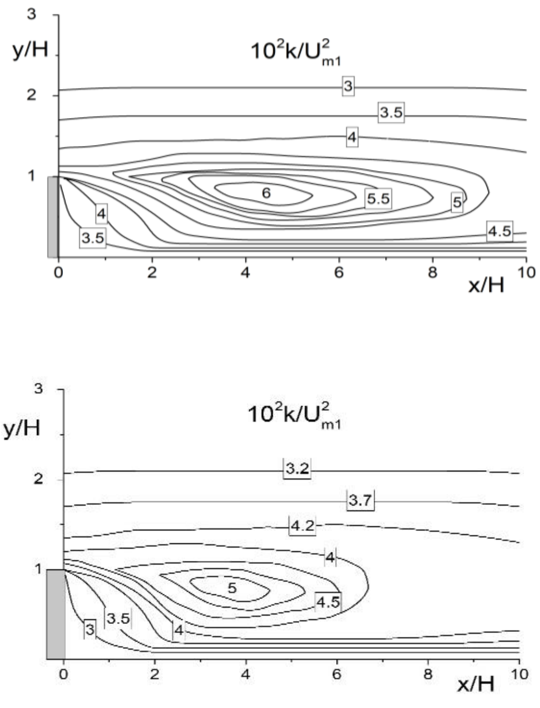
Constant values of variables are set at the pipe inlet, and soft boundary conditions are set at the outlet.

Numerical method for solving the problem. The numerical solution is obtained using a control volume method on a staggered grid. The algorithm for solving the system of equations (1)-(6) in the variables "velocity-pressure components" is described in detail in the work [[7], [8], [9]].

Numerical calculations are obtained using our software.

Discussion of the calculated data

Flow structure in a pipe with a sudden expansion. Figure 2 shows the contour lines of turbulent kinetic energy for Newtonian (NF) and viscoplastic fluids (NNF). The contour lines show the structure of the mixing layer and the development of turbulent kinetic energy in the flow region. From the edge of the sharp expansion area, the mixing layer separates and develops to the point of re-attachment of the flow (see Figure 2). The separation zone for the Newtonian fluid reaches x/H=9.3, and for the viscoplastic fluid - x/H=6.7, i.e. the separation zone is reduced in the viscoplastic fluid. The maximum level of turbulent kinetic energy is achieved in the mixing layer (k/U_{m1}^2 = 0.06). The region with maximum TKE values (k/U_{m1}^2 = 0.06) is located in the downstream position (x/H ≈ 5, y/H ≈ 0.6) for Newtonian fluid and (k/U_{m1}^2 = 0.045) for viscoplastic fluid in the position (x/H ≈ 4, y/H ≈ 0.5) (see Figure 2).



a)

b)

Figure 2 - Plots of the kinetic energy of turbulence in NF (a) and NNF (b) fluids behind a sudden expansion of a pipe. Re = 10^4, Re_H =2600, Pr = 42, Bn_1 = 0.007

Fig. 3 shows the change in the maximum axial velocity along the longitudinal coordinate in the recirculation zone of the flow behind a sudden expansion of the pipe.

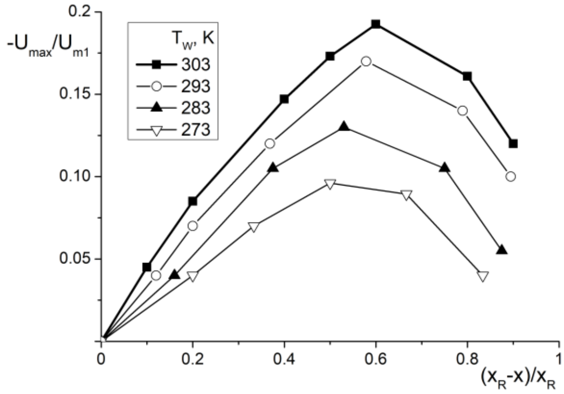


Figure 3 - Change in axial velocity along the length of the recirculation zone in a pipe with a sudden expansion. Re = 10^4, Re_H =2600, Pr = 42, Bn_1 = 0.007

The calculations were performed at the inlet temperature T_1=303 K and different values of the wall temperature T_w. The calculated data at T_w=303 K correspond to the turbulent flow of a Newtonian fluid (NF). In this mode, the maximum value of the velocity is $-U_{max}/U_{m1}$ ≈ 0.2. For a viscoplastic fluid at T_w=273 K, the maximum value of the velocity is $-U_{max}/U_{m1}$ ≈ 0.1. It can be said that the appearance of the viscoplastic property of the fluid suppresses the intensity of the circulation flow compared to a Newtonian fluid (see Figure 3).

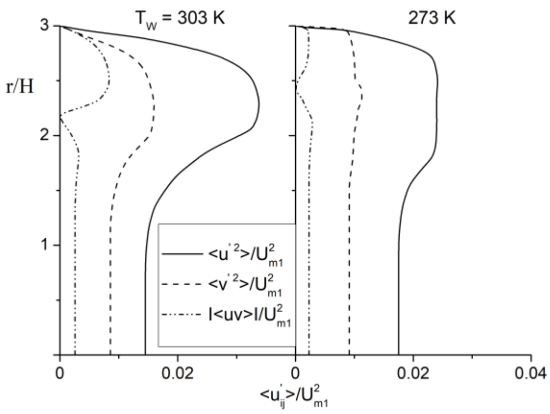


Figure 4 - Reynolds stress profiles of NF (T_w = 303 K) and NNF (T_w = 273 K) fluids at x/H = 4. Re = 10^4, Re_H =2600, Pr = 42, Bn_1 = 0.007

Figure 4 shows the distributions of Reynolds stresses for Newtonian (Tw = 303 K) and viscoplastic (Tw = 273 K) fluids at x/H = 4. The highest value of Reynolds stresses is achieved in the shear layer of mixing of Newtonian and non-Newtonian fluids. These data are in agreement with the results of separated flows of Newtonian fluid after a step [6]. The highest value of the ratio of axial velocity fluctuations to radial ones for NF fluid is up to 2 times and for NNF fluid up to 2.6 times.

The length of the circulation zone characterizes the intensity of the vortex in a pipe with a sharp expansion (see Figure 5). The Reynolds number of the flow has little effect on the position of the zone attachment point and the maximum heat transfer for Newtonian and non-Newtonian fluids for all calculations of wall temperatures and flow rates (up to 7%). The position of the flow reattachment point and the maximum heat transfer for Newtonian and non-Newtonian fluids are slightly affected by the Reynolds number (up to 7%) for all calculations of wall temperatures and flow rates. The difference between the flow reattachment point and the maximum heat transfer between Newtonian and non-Newtonian fluids does not exceed 5%. The flow reattachment point and the position of maximum heat transfer are located upstream for a non-Newtonian fluid compared to the position of the reattachment point in Newtonian fluids [[6], [17], [18], [19]]. The maximum difference reaches 10% for a non-Newtonian fluid at Tw = 273 K.

The length of the separation zone and the position of the maximum heat transfer are affected by the wall temperature. For a non-Newtonian fluid, the length of the recirculation zone is reduced to 66% compared to the length of a Newtonian fluid and is xR/H ≈ 6.5–6.8 at Tw = 273 K.

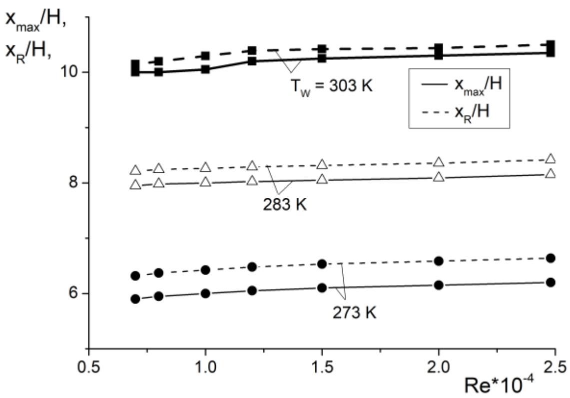
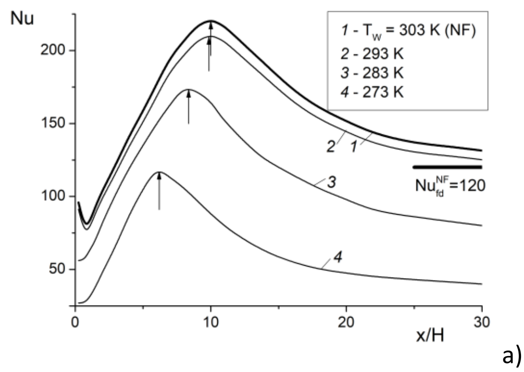


Figure 5 - Recirculation length x_R and maximum heat transfer point x_{max} as a function of wall temperature. Re = 10^4, Re_H =2600, Pr = 42, Bn_1 = 0.007

Heat transfer. Figure 6 shows the changes in the Nusselt number Nu = hH/λ_{W1} along the axial coordinate (a) and the effect of the Reynolds number Re on the maximum heat transfer (b). Here, h is the heat transfer coefficient, Numax is the maximum Nusselt number, and Nu_{fd} is the Nusselt number for a fully developed Newtonian flow in a pipe without expansion.

The Nusselt number is found from the step height $Nu = -(\partial T/\partial y)_W H/(T_W - T_m)$. The flow reattachment points are indicated by arrows in Fig. 6a. As can be seen from Fig. 6a, the changes in heat transfer along the pipe length for NF (line 1) and NNF (lines 2–4) are qualitatively similar. There is no local minimum for NNF in the corner part of the step. At Tw = 293 K, the change in heat transfer (curve 2) and the length of the recirculation zone of NNF are practically the same as the NF data. The faster manifestation of non-Newtonian properties of the NNF fluid causes a decrease in the wall temperature and confirms the above-mentioned calculated data. A decrease in the intensity of heat transfer of turbulent flow in a pipe with sudden expansion can be noted for NNF. The maximum heat transfer values coincide with the location of the flow reattachment point. For NNF at Tw = 273 K, the maximum heat transfer is shifted by up to 70% (line 4).



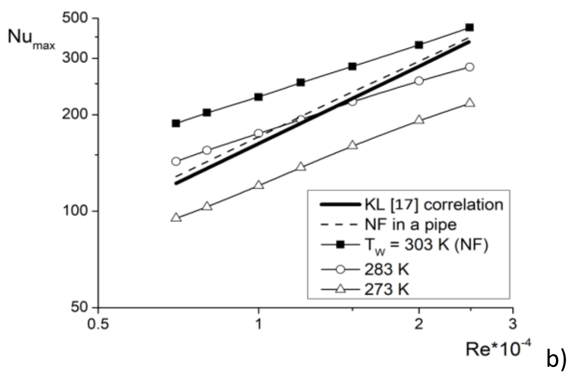


Figure 6 - Dependence of maximum heat transfer on Nusselt (a) and Reynolds (b) numbers in NF and NNF turbulent flows.
Re = 10^4, Re_H =2600, Pr_1 = 42, Bn_1 = 0.007

Similar data were obtained for the maximum values of the Nusselt number depending on the Reynolds number (see Fig. 6b). The bold line is the calculated data using the formula $Nu = 0.023 Re^{0.8} Pr^{0.4}$ [20] for a stabilized turbulent flow of a Newtonian fluid in a pipe without expansion.

The authors' calculated data for a fully developed turbulent Newtonian fluid in a pipe are shown as a dashed line. An increase in the Reynolds number leads to an increase in heat transfer for turbulent NF, as well as for NNF. For a turbulent flow of a Newtonian fluid, the maximum value of heat transfer is higher (dashed line) than for turbulent flows of a non-Newtonian fluid (T_w = 283 and 273 K). It can be noted that the slope of the well-known experimental heat transfer formula [20] (bold line) and the authors' calculated data for a pipe without expansion differ from the data for NF and NNF after a sudden expansion of the pipe.

Conclusion

Calculated data are obtained for the turbulent non-isothermal flow of non-Newtonian fluid in a pipe with abrupt expansion. An elliptic model of Reynolds stress relaxation is used to simulate turbulent kinetic energy. The calculations show the occurrence of completely stagnant flow in the near-wall region of a pipe for non-Newtonian fluid, where the value of $U \approx 0$ ($y/R \leq 0.4$ at $x/H = 15$). The stagnation zone occurs at $x/H > 8$ with the Bingham number Bm=17. In the shear mixing layer of the isothermal non-Newtonian fluid, a significant decrease in the turbulence level (up to 50%) is obtained. In the core zone, the turbulence level of the non-Newtonian fluid is up to 15% higher than that of the Newtonian fluid. The Bingham number affects the value of the maximum negative velocity in the separation region ($-U_{max}/U_{m1} \approx 0.075$ with Bm=17).

For a non-Newtonian fluid, the vortices in the recirculation zone after a sudden expansion are less intense than for a Newtonian fluid.

For turbulent flows of non-Newtonian and Newtonian fluids, the heat transfer distributions are qualitatively similar. The values of the location of the flow attachment point and the maximum heat transfer are also close, the difference does not exceed 10%. The length of the recirculation zone and the position of the maximum heat transfer are strongly affected by the wall temperature. For a non-Newtonian fluid, the length of the recirculation zone is shorter by up to 66% than for a Newtonian fluid. The flow attachment point of a non-Newtonian fluid is located higher than that of a Newtonian fluid. Heat transfer in the recirculation zone of a non-Newtonian fluid is two times lower than for a Newtonian fluid. The length of the recirculation zone is shorter by up to 40% for a non-Newtonian fluid, compared to a Newtonian fluid.

Conflicts of interest. On behalf of all authors, the corresponding author states that there is no conflict of interest.

CRediT author statement. U. Zhapbasbayev: contributed to the conceptualization, investigation, methodology, and original draft preparation; **D. Bossinov:** contributed to the methodology, visualization, and writing—review and editing.

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Құбырдың кенеттен кеңейетін аймағындағы турбуленттік изотермиялық емес ағынның құрылымы

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ТҮЙІНДЕМЕ
Мақалада Ньютондық емес сұйықтықтың турбулентті изотермиялық емес ағынының математикалық моделі қарастырылады. Құбырға кіргенде сұйықтық Ньютондық болып табылады және температураның төмендеуіне байланысты тұтқырлық пен аққыштық беріктігінің жоғарылауына байланысты ол Ньютондық емес сұйықтыққа айналады. Турбулентті қозғалыс пен жылу алмасу теңдеулер жүйесі жылдамдық пен қысымның айнымалы құрамдас бөліктерінде сандық басқару көлемі әдісімен шешіледі. Есептеулер нәтижесінде кенет кеңеюі бар құбырдағы Ньютондық емес сұйықтықтың изотермиялық емес қозғалысының орташа және пульсациялық сипаттамалары алынды. Есептеулер рециркуляция аймағының құрылымының күрт төмендеуін және Бингам санының Bn ұлғаюымен оның параметрлерінің төмендеуін көрсетеді. Бұл аймақта Ньютон сұйықтығы үшін –Umax/Um1 ≈ 0,2-ге тең орташа жылдамдықтың максималды теріс мәні Bingham санында Bn = 17 –Umax/Um1 ≈ 0,1-ге дейін төмендейді. Ньютондық емес сұйықтық ағынының турбуленттік сипаттамаларының төмендеуі Бингем санының жоғарылауымен де байқалады. Турбуленттік Ньютондық емес және Ньютондық сұйықтықтардың ағыс аймағындағы жылу алмасу сипаттамалары сапалық жағынан ұқсас. Ньютондық емес сұйықтықтың ағындық қосылысының орны және максималды жылу беруі 10% аспайды. Ньютондық емес сұйықтықтың рециркуляция аймағының ұзындығы Ньютон сұйықтығымен салыстырғанда 66%-ға дейін қысқа.

Түйін сөздер: изотермиялық емес турбулентті ағын, тұтқыр пластикалық сұйықтық, кенеттен кеңеюі бар құбыр ағысының рециркуляция аймағы.

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Структура турбулентного неизотермического течения в трубе с внезапным расширением

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АННОТАЦИЯ
В статье рассматривается математическая модель турбулентного неизотермического течения неньютоновской жидкости. На входе жидкость является ньютоновской и из-за понижения температуры становится неньютоновской за счет увеличения вязкости и предела текучести. Система уравнений турбулентного движения и теплопереноса решается численным методом контрольного объема в переменных компонентах скорости и давления. В результате расчетов получены средние и пульсационные характеристики неизотермического движения неньютоновской жидкости в трубе с внезапным расширением. Расчеты показывают резкое сокращение структуры зоны рециркуляции и уменьшение ее параметров с ростом числа Бингама Bn. В этой зоне максимальное отрицательное значение средней скорости, равное –Umax/Um1 ≈ 0,2 для ньютоновской жидкости, уменьшается до –Umax/Um1 ≈ 0,1 при числе Бингама Bn = 17. Также наблюдается уменьшение турбулентных характеристик течения неньютоновской жидкости с ростом числа Бингама. Характеристики теплообмена в области течения турбулентных неньютоновской и ньютоновской жидкостей качественно подобны. Расположение присоединения потока и максимального теплообмена неньютоновской жидкости не превышает 10%. Длина зоны рециркуляции неньютоновской жидкости короче до 66% по сравнению с ньютоновской жидкостью.

Ключевые слова: неизотермическое турбулентное течение, вископластическая жидкость, зона рециркуляции потока в трубе с резким расширением.

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