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Isothermal laminar flow of non-newtonian fluid with yield stress in a pipe

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ABSTRACT

This paper considers the development of an isothermal laminar flow of viscoplastic fluid with yield stress in a pipe. A characteristic feature of such a flow is the formation of a non-deformable region in which the fluid behaves like a solid. This phenomenon significantly complicates the numerical solution of the equations of viscoplastic fluid flow, since traditional methods cannot adequately describe the behavior of the fluid in this region. The novelty of this work resides in the application of the effective molecular viscosity methodology and the Bingham-Papanastasiou model, which made it possible to perform an end-to-end calculation of the isothermal flow taking into account the non-deformable region. In the course of the calculations, the velocity and pressure distributions were derived for Reynolds numbers from 71.2 to 740.8 and Bingham numbers in the range from 1.225 to 17.01. An increase in the Reynolds number to Re = 740.8 and a decrease in the Bingham number to Bn = 1.225 lead to a reduction in the region with maximum velocities and a change in the input axial velocity distribution. The radial profiles of the axial velocity remain the same in all cross-sections from z/R = 10 to z/R = 40, which indicates the establishment of a steady-state flow regime of viscoplastic fluid, in which a constant velocity core is formed in the cross-section of the pipe.

Keywords: viscoplastic fluid flow, effective molecular viscosity approach, yield stress, bingham-papanastasiou model.

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Introduction

Non-Newtonian fluids with yield stress are encountered in various industrial processes, such as the transportation of paraffinic crude oil in underground and underwater pipelines in offshore fields, as reported by several authors [[1], [2], [3], [4]].

Non-Newtonian fluids have a natural time (fluid time scale). The relaxation time of viscoelastic fluids, the time scale of thixotropic fluids, and the time scale of viscoplastic fluids (the ratio of plastic viscosity to yield stress) are examples of non-Newtonian fluid time scales.

Several theoretical studies focus on the flow of Bingham non-Newtonian fluids [[5], [6]]. The initial boundary value problem related to Bingham fluid motion is examined by Luckring, who proves the existence and uniqueness of a strong solution under specific assumptions about the data [5]. Luckring also shows that the solution exists globally over time when the data is small and approaches a periodic solution when the external force is time-periodic.

The mathematical model describing three-dimensional steady Bingham fluid flow in a confined region under threshold slip boundary conditions is discussed by Baranovskii [6]. It is assumed by Baranovskii that the flows can slip along solid surfaces when shear stresses reach a certain critical value. A weak formulation of this problem is developed using the variational inequality approach. The necessary conditions for the existence of weak solutions are determined, along with the corresponding energy estimates [6].

The challenge in numerically modeling viscoplastic fluid flow lies in the presence of a rigid (undeformed) region within the flow field. In the literature, two categories of methods have been suggested to tackle this mathematical problem. The first approach, commonly referred to as the regularization method, is extensively utilized by many researchers and involves representing the effective molecular viscosity as a continuous function [[7], [8], [9], [10], [11], [12], [13], [14]]. The exponential formula introduced by Papanastasiou represents the most widely adopted variant of the regularization method [8]. This methodology is straightforward to implement, as the regularized equation transforms the mathematical problem into a viscous one. The criterion for determining whether the flow region is deformable or undeformable becomes irrelevant, since the deformation rate tensor approaches zero, as noted by authors [[9], [10]]. Consequently, the rigid part of viscoplastic fluid flow can be estimated with sufficient accuracy.

Another method, conceptually more intricate, is derived from the theory of variational inequalities and takes advantage of Lagrange multiplier techniques. For a thorough and precise mathematical examination of the associated variational inequalities, refer to Duvaut and Lions [15]. This method simplifies the problem by transforming it into the minimization of an extended Lagrangian functional. The resulting saddle-point problem has been addressed by various researchers using a Uzawa-type algorithm [[16], [17], [18]]. The primary benefit of the extended Lagrange method lies in its integration of the constitutive equation, which facilitates the identification of undeformed regions through a zero strain rate tensor and provides a clear distinction between deformable and non-deformable regions. Computational and theoretical studies include analyses of lid-driven cavity flow by several authors [[19], [20], [21]]. Flow around a cylinder has been investigated by Roquet et al., [22]. Additionally, flow in converging geometries has been studied by Coupez et al., [23].

To study the non-isothermal flow of viscoplastic fluid within the pipe, the aforementioned method is discussed in detail in [24]. A numerical method was developed to solve the system of motion and energy equations using a TVD scheme. Special attention is given to the velocity-pressure problem, where the Bingham model is incorporated (without a regularization procedure) using Lagrange multiplier methods and extended Lagrange/Uzawa methods by Vinay et al., [24]. The results obtained for the stationary solution highlight the impact of temperature variations on the flow pattern, particularly concerning deformable and non-deformable regions. Specifically, in pipe flow, the temperature field varies along the flow direction.

It should be noted that the solution method is labor-intensive and has been applied in only a few studies of viscoplastic fluids [24].

The regularization method by Papanastasiou is widely applied to solve various practical problems, such as the non-isothermal flow of Bingham fluid in cases of sudden pipe expansion, as noted by authors [[25], [26]].

As highlighted in the review, multiple facets of the motion and heat transfer of Bingham fluids have been explored. However, there is a limited number of studies offering an in-depth analysis of the isothermal flow of viscoplastic fluids.

The objective of this study is to examine the isothermal flow of viscoplastic fluids through a numerical solution of a system of motion equations, using a regularization method by Papanastasiou and an effective molecular viscosity approach by Bird et al. [27].

Isothermal laminar flow of viscoplastic fluid Problem statement

An isothermal flow of viscoplastic fluid enters a pipe with an average inlet velocity (see Fig. 1). At specific yield stress values, a stagnation zone forms near the pipe wall, where the flow velocity becomes zero. The Reynolds (Re) and Bingham (Bn) numbers are derived from the characteristics of viscoplastic fluid at the pipe inlet. The pipe inner diameter is D = 0.05 m and the pipe length is L = 1 m, resulting in a length-to-radius ratio of L/R = 40.

The inlet profile of axial velocity transforms, and in a certain section, a velocity distribution corresponding to the flow of Bingham fluid is established. Our task is to define the establishment of the Bingham fluid profile as a dependent variable of the Reynolds and Bingham numbers.

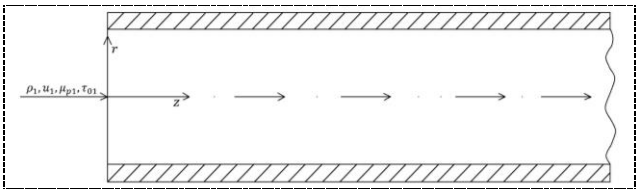

Figure 1 - Diagram of the isothermal flow of viscoplastic fluid within pipe

Bingham-Papanastasiou model.

Based on the rheological behavior of viscoplastic fluids, the effective molecular viscosity can be represented as outlined by various authors [[27], [28], [29], [30], [31]]:

$$\mu_{eff} = \begin{cases} \mu_p + \tau_0 \left|\dot{\gamma}\right|^{-1}, & \text{if } \left|\tau\right| \quad \tau_0 \\ \infty, & \text{if } \left|\tau\right| \le \tau_0 \end{cases} \tag{1}$$

The expressions in formula (1) are provided by Pakhomov et al., [32].

However, due to mathematical complexities, Eq. (1) cannot be used without regularization. For this purpose, the formula presented by Papanastasiou is employed [8]. In this scenario, the effective molecular viscosity is constrained as the shear rate approaches zero $\left|\dot{\gamma}\right| \to 0$, as observed by Pakhomov et al. [32]:

$$\mu_{eff} = \mu_p + \tau_0 \frac{\left[1 - \exp\left(-10^3 \left|\dot{\gamma}\right|\right)\right]}{\left|\dot{\gamma}\right|} \tag{2}$$

Fundamental equations of heat transfer.

The equations governing mass and heat transfer of fluid can be expressed in non-dimensional form within a cylindrical coordinate system, as presented by authors [[28], [31]]:

$$\frac{\partial U}{\bar{z}} + \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} (\bar{r} V) = 0 \tag{3}$$

$$U \frac{\partial U}{\partial \bar{z}} + \quad \frac{\partial U}{\partial \bar{r}} = -\frac{\partial P}{\partial \bar{z}} + \frac{1}{\mathrm{Re}} \left[\frac{\partial}{\partial \bar{z}} \left(2\mu_{eff} \frac{U}{-} \quad + \right. \right. \left. \left. \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\mu_{eff} \bar{r} \left(\frac{\partial U}{\partial \bar{r}} + \frac{\partial}{\partial \bar{z}} \right) \right) \right] \tag{4}$$

$$U \frac{\partial V}{\partial \bar{z}} + V \frac{\partial V}{\partial \bar{r}} = -\frac{\partial P}{\partial \bar{r}} + \frac{1}{\mathrm{Re}} \left[\frac{\partial}{\partial \bar{z}} \left(\mu_{eff} \left(\frac{\partial V}{\partial \bar{z}} + \frac{\partial U}{\partial \bar{r}} \right) \right) - \frac{2\mu_{eff} V}{\bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(2\bar{r} \mu_{eff} \frac{\partial V}{\partial \bar{r}} \right) \right] \tag{5}$$

here $\bar{z} = z / R; \bar{r} = r / R; U = u / u_1; V = v / u_1; P = p / \rho u_1^2;$

Re is the Reynolds number.

The plastic viscosity and yield stress coefficient dependences on temperature are provided by Pakhomov et al., [32].

Boundary conditions.

No slip on the pipe wall, as described by Pakhomov et al., [32]:

$$\bar{r} = 1 : U = V = 0 \tag{6}$$

Symmetry on the pipe axis, as described by Pakhomov et al., [32]:

$$\bar{r} = 0 : \frac{\partial U}{\partial \bar{r}} = \frac{\partial V}{\partial \bar{r}} = 0 \tag{7}$$

Constant velocity at the pipe inlet, as described by Pakhomov et al., [32]:

$$\bar{z} = 0 : U = 1, V = 0 \tag{8}$$

Neumann boundary at the pipe outlet, as described by Pakhomov et al., [32]:

$$\bar{z} = L / R : \frac{\partial U}{\partial \bar{z}} = \frac{\partial V}{\partial \bar{z}} = 0 \tag{9}$$

Numerical implementation.

The numerical results are obtained using a control volume method applied on a staggered grid. The algorithm for solving the system of Eq. (3)-(6) in terms of the "velocity-pressure components" is detailed by Pakhomov et al., [32].

The equations were discretized using the finite volume method on a staggered grid. The pressure field p and the velocity values u, and v, each one had its own unique grids, resulting in individual control volumes. The power-law scheme was applied to the convective terms in the differential equations [33]. Second-order central difference methods were used for diffusive flows [32].

The SIMPLE algorithm was used to solve Eq. (3)-(5), with each iteration involving the following steps.

All numerical predictions are conducted using an "in-house" code.

To verify the calculations, known results for laminar flow of Bingham fluid can be utilized. Figure 2 displays the computed data for the radial distribution of non-dimensional axial velocity (a) and dynamic viscosity (b) across the section of a pipe [34].

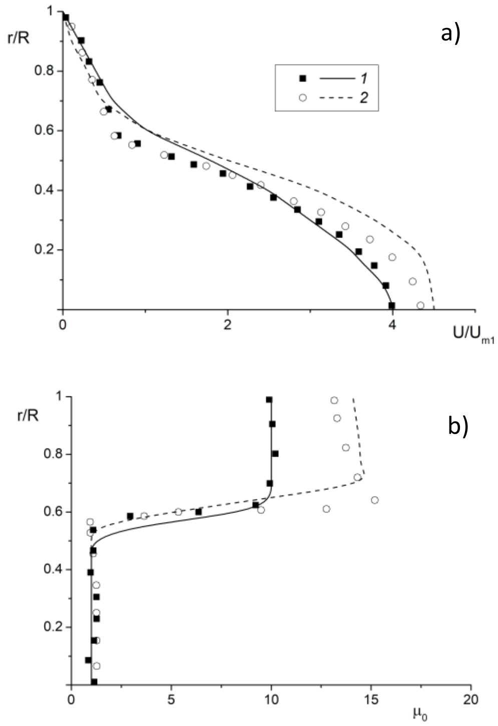

Figure 2 – Non-dimensional axial velocity (a) and dynamic viscosity (b) profiles across the section of a pipe. The lines illustrate the authors' calculations, while the points represent calculated data [34]:
1 – Bn = 0; 2 – Bn = 5; Sc = 10; q/R = 0.1; R_1/R = 0.55; μ_P/μ_1 = 10; Re = $\rho_1 R u_1/\mu_1$ = 1000

The computations were performed for two mixed fluids: a Newtonian fluid flowing in the center of a pipe ($_1 / R \le 0.55$) and a Bingham fluid ring introduced in the wall-adjacent region $(R_2 / R = (R_1 + q) / R = 0.65 - 1)$. The intermediate mixing layer's thickness between the Newtonian fluid and Bingham fluid is $q / R = 0.1$. Notably, in this case, the mathematical model was adapted by incorporating a diffusion equation with a specified Schmidt number $Sc = \mu_1 / (\rho_1 D_S) = 10$ [34]. In this context, the subscripts "1" and "2" denote the Newtonian fluid and Bingham fluid, respectively, while D_S represents the coefficient of molecular diffusion. Comparisons were conducted between the isothermal laminar flow regime of the Newtonian fluid (1) and the viscoplastic fluid characterized by a specified Bingham number (Bn). The isothermal laminar flow regime of the Newtonian fluid (1) was compared to the viscoplastic fluid characterized by a specified Bingham number of $Bn = \tau_0 R / (\mu_1 u_1) = 5$ (2). A notable quantitative consistency was observed between our numerical results and the findings reported in [34].

Discussion and Results

The simulations were performed for a pipe with a length L = 1 m and a diameter D = 2R = 0.05 m (L/R = 40). The calculations were carried out in a pipe with a diameter of D = 2R = 0.05 m and a length of L = 1 m (L/R = 40). The average flow velocity at the pipe inlet u_1 varied from 0.05 to 0.20 m/s. The paraffinic oil density is constant and equal to 850 kg/m^3. The Reynolds and Bingham numbers vary: range from 71.2 to 740.8 and $Bn = \tau_{0w} 2R / (\mu_{pw} u\ )$ ranges from 0.17 to 17.01.

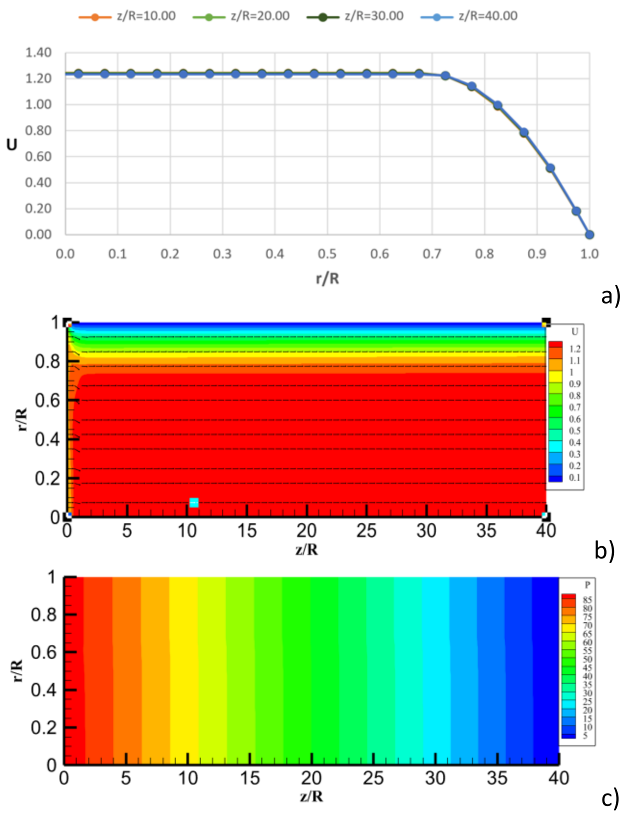

Figure 3 – Radial profile of axial velocity (a), velocity vector contours (b) and pressure (c) under the operating conditions: u_1 = 0.10 m/s, μ_{p1} = 0.05974 $Pa \cdot s$, τ_{01} = 2.03286 Pa, Re = 71.2, and Bn = 17.01

Figure 3 shows the calculated data for an average velocity of u_1 = 0.10 m/s, plastic viscosity of μ_{p1} = 0.05974 $Pa \cdot s$, yield stress of τ_{01} = 2.03286 Pa, Reynolds numbers Re = 71.2, and Bingham numbers Bn = 17.01.

The radial distributuions of axial velocity U exhibit a core of constant values (see Figure 3a), characteristic of viscoplastic fluid flow. The core of constant velocities U occupies a radius from r/R = 0 to r/R = 0.67, starting from the section z/R = 10 to z/R = 40, i.e. the establishment of a radial profile of axial velocity across the length of the pipe takes place.

The velocity vector contours clearly demonstrate the rapid transformation of the inlet profile of axial velocity U and the establishment of viscoplastic fluid flow (see Figure 3b).

The pressure contours show the distribution of P along the pipe length (Figure 3c). The pressure remains constant across the pipe's cross-section and drops throughout its length. The value of dimensionless pressure is equal to P = 89 or p = 765.5 Pa at the beginning of the pipe and decreases along the pipe length. The pressure difference of Δp = 765.5 Pa ensures the movement of visco-plastic fluid along the length of a pipe.

The results derived from the calculations under operating parameters u_1 = 0.10 m/s, μ_{p1} = 0.02438 $Pa \cdot s$, τ_{01} = 0.11937 Pa, Re = 174.3, and Bn = 2.45 are presented in Figure 4. The radial profiles of axial velocity U have a core of constant values along the radius from r/R = 0.0 to r/R = 0.43, starting from the section z/R = 2 to z/R = 40 along the pipe length (see Figure 4a). A decrease in the Bingham number Bn = 2.45 and a growth in the Reynolds number Re = 174.3, results in to a reduction in the core length of constant data of U and, accordingly, a growth in a magnitude of the axial velocity (see Figure 4a). It is evident that the axial velocity profiles are established starting from the section z/R=2 and correspond to the velocity distribution of a Bingham fluid (see Figure 4a).

The velocity vector contours U clearly illustrate the establishment of a steady flow of viscoplastic fluid and the location of the core of constant velocities along the radius and length of a pipe (see Figure 4b).

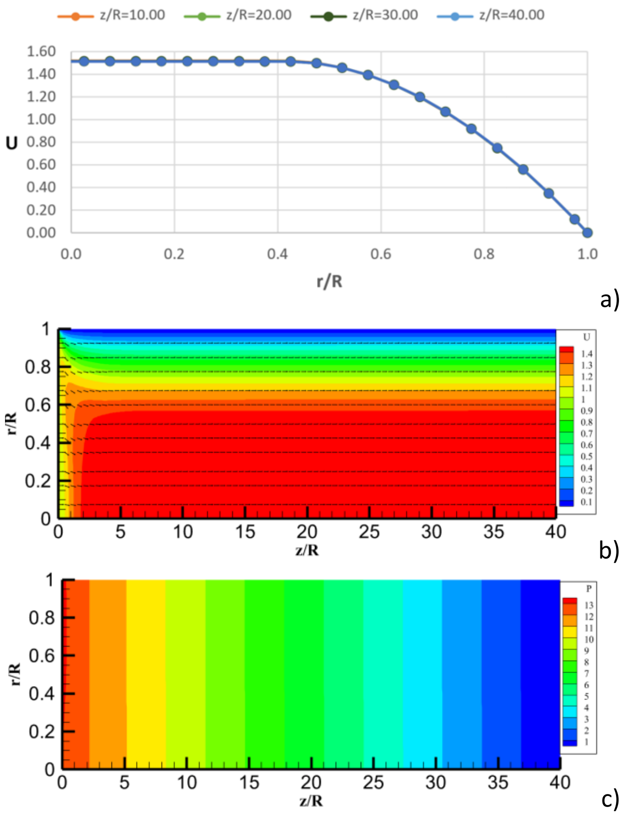

Figure 4 – Radial profile of axial velocity (a), velocity vector contours (b) and pressure (c) under the operating conditions: u_1 = 0.10 m/s, μ_{p1} = 0.02438 $Pa \cdot s$, τ_{01} = 0.11937 Pa, Re = 174.3, and Bn = 2.45

The pressure contours P show a reduction in their values throughout the length of the pipe (Figure 4c). The Bingham number is Bn = 2.45, almost 7 times less than in the previous case. This shows a decrease in the effect of plastic viscosity and yield stress on hydraulic flow resistance. The pressure loss is Δp = 114.8 Pa, which is lower than the earlier situation (Figure 4c).

The computed results at operating parameters: u_1 = 0.20 m/s, μ_{p1} = 0.05974 $Pa \cdot s$, τ_{01} = 2.03286 Pa, Re = 142.2, and Bn = 8.51 were presented in Figure 4. As observed from the radial distribution of axial velocity U, the core of constant values of U has the same value both in the radial direction and along the pipe length, indicating the establishment of the flow of viscoplastic fluid (see Figure 5a). The region with the core of constant values of U is located from z/R = 1 to z/R = 40 throughout the length of the pipe (Figure 5a).

Velocity vector contours show establishment of the axial velocity profile U of viscoplastic fluid throughout the length of the pipe (Figure 5b).

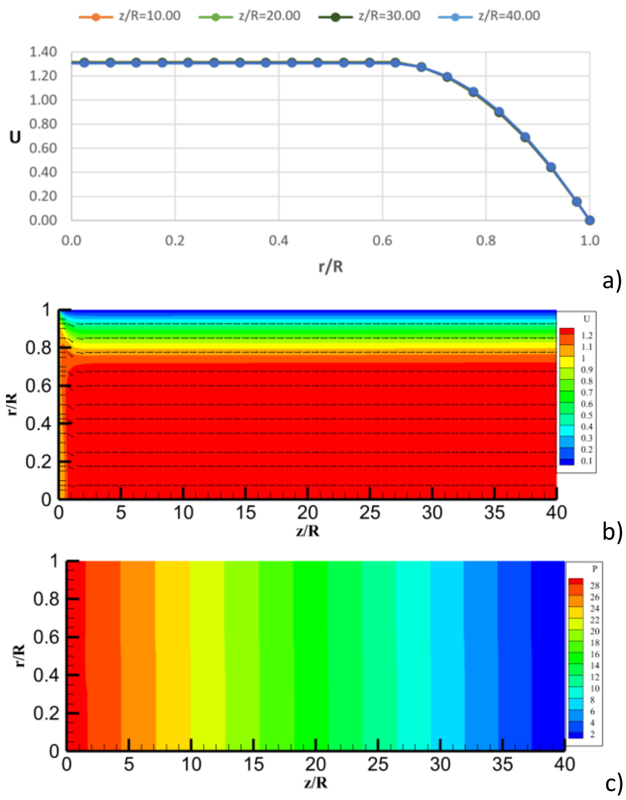

Figure 5 – Radial profile of axial velocity (a), velocity vector contours (b) and pressure (c) under the operating conditions: u_1 = 0.20 m/s, μ_{p1} = 0.05974 $Pa \cdot s$, τ_{01} = 2.03286 Pa, Re = 142.2, and Bn = 8.51

The contours of the dimensionless pressure P indicate a reduction in hydraulic loss of viscoplastic fluid flow (see Figure 5c). The pressure reduction is Δp = 986 Pa and ensures laminar viscoplastic fluid flow in a pipe (see Figure 5c).

Figure 6 shows the calculated data at operating parameters: u_1 = 0.20 m/s, μ_{p1} = 0.02438 $Pa \cdot s$, τ_{01} = 0.11937 Pa, Re = 740.8, and Bn = 1.225. The increase in the Reynolds number to Re = 740.8 and the decrease in the Bingham number to Bn = 1.225 lead to a reduction in the core of maximum velocities U (see Figure 6a). The appearance of the initial section of the transformation of the inlet axial velocity profile can be seen. The radial distributions of axial velocity U exhibit the same shape in all cross-sections between z/R = 6 and z/R = 40. This corresponds to Bingham fluid flow with a constant core velocity U in the pipe cross-section and indicates the establishment of a steady-state laminar flow regime of viscoplastic fluid (Figure 6a).

The contours of the velocity vectors depict a detailed picture of flow through the cross-section and throughout the pipe's length (Figure 6b). One can observe the flow core with a constant value of axial velocity U and a decrease in its value to zero at the wall. The Bingham number Bn = 1.225 leads to a reduction in head loss relative to the earlier case. The pressure loss Δp is 238 Pa, which was sufficient for viscoplastic fluid to flow throughout the pipe's length (Figure 6c).

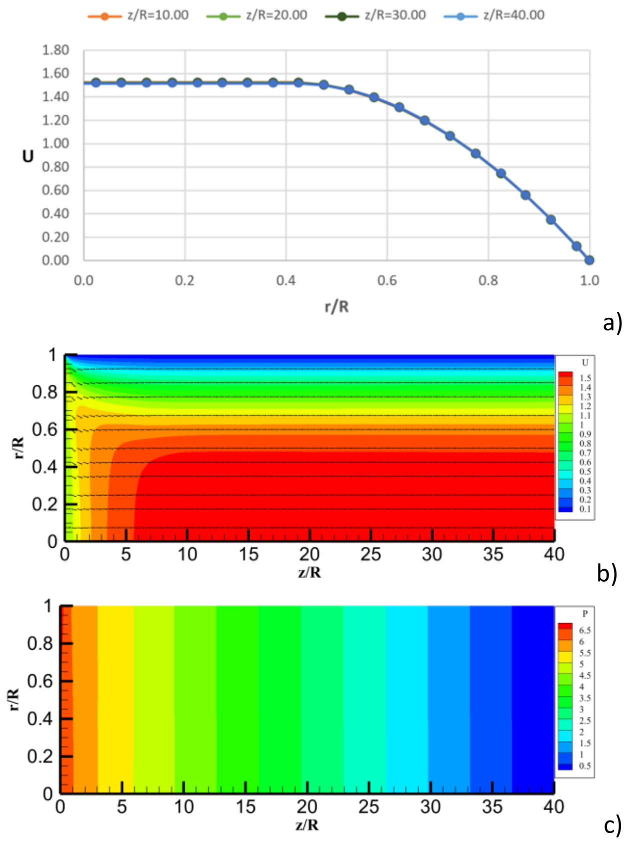

Figure 6 – Radial profile of axial velocity (a), velocity vector contours (b) and pressure (c) under the operating conditions: u_1 = 0.20 m/s, μ_{p1} = 0.02438 $Pa \cdot s$, τ_{01} = 0.11937 Pa, Re = 348.6, and Bn = 1.225

Conclusions

The paper discusses the findings of the study on Laminar isothermal viscoplastic fluid flow in the pipe, taking into account yield stress and plastic viscosity. The calculated data were obtained by numerically solving the system of equations for viscoplastic fluid flow. The computations determined the effect of the Bingham number and Reynolds number on the axial velocity profiles and pressure distribution. The regions of constant axial velocity values are shown depending on the values of the Bingham number and Reynolds number. The larger the Bingham number and the lower the Reynolds number, the longer the core of constant velocity in the cross-section of the pipe. These findings contribute to a deeper understanding of viscoplastic flow dynamics, with significant implications for various engineering applications and fluid transport systems. In further studies, the molecular effective viscosity approach and the regularization method will be used to calculate viscoplastic fluid flow in various practical applications.

Conflicts of interest. On behalf of all authors, the corresponding author states that there is no conflict of interest.

CRediT author statement. **T.Bekibayev**: Software, Methodology; **G. Ramazanova:** Visualization, Writing; **D. Bossinov**: Translating, Editing; **Muhammad Noorazlan**: Reviewing.

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Құбырдағы аққыштық шегі бар ньютондық емес сұйықтықтың изотермиялық ламинарлы ағыны

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Қабылданды: *4 желтоқсан 2024*

ТҮЙІНДЕМЕ
Осы жұмыста құбырдағы аққыштық шегі бар тұтқыр пластик сұйықтықтың изотермиялық ламинарлық ағынының дамуы қарастырылады. Мұндай ағынға тән ерекшелік - деформацияланбайтын аймақ пайда болады, онда сұйықтық қатты зат сияқты әрекет етеді. Бұл құбылыс тұтқыр пластикалық сұйықтық ағынының теңдеулерін сандық шешуді едәуір қиындатады, өйткені дәстүрлі әдістер бұл аймақтағы сұйықтықтың әрекетін жеткілікті түрде сипаттай алмайды. Жұмыстың жаңалығы тиімді молекулалық тұтқырлық әдіснамасын және Бингам-Папанастасиу моделін қолдану болып табылады, бұл деформацияланбайтын аймақты ескере отырып, изотермиялық ағынды түпкілікті есептеуге мүмкіндік берді. Есептеулер арқылы 1.225-тен 17.01-ге дейінгі Бингам сандары мен 71.2-ден 740.8-ге дейінгі Рейнольдс сандары үшін жылдамдық пен қысым үлестірімдері алынды. Рейнольдс санының Re = 740.8-ге дейін өсуі және Бингам санының Bn = 1.225-ке дейін төмендеуі аймақтың максималды жылдамдықпен қысқаруына және аксиалды жылдамдықтың кіре берістегі таралуының өзгеруіне әкеледі. Аксмалды жылдамдықтың радиалды профильдері z/R = 10-дан z/R = 40-қа дейінгі барлық көлденең қималарда бірдей болып қалады, бұл құбырдың көлденең қимасында тұрақты жылдамдық ядросы пайда болатын тұтқыр пластикалық сұйықтық ағынының тұрақты режимінің орнатылғанын көрсетеді.

Түйін сөздер: тұтқыр пластикалық сұйықтық ағыны, тиімді молекулалық тұтқырлық аппараты, аққыштық шегі, Бингем-Папанастасиу моделі.

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Изотермическое ламинарное течение неньютоновской жидкости с пределом текучести в трубе

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АННОТАЦИЯ
В настоящей работе рассматривается развитие изотермического ламинарного течения вязкопластичной жидкости с пределом текучести в трубе. Характерной особенностью такого течения является образование недеформируемой области, в которой жидкость ведет себя как твердое тело. Это явление значительно усложняет численное решение уравнений течения вязкопластичной жидкости, так как традиционные методы не могут адекватно описать поведение жидкости в этой области. Новизна работы заключается в применении методологии эффективной молекулярной вязкости и модели Бингама-Папанастасиу, что позволило провести сквозной расчет изотермического течения с учетом недеформируемой области. В ходе расчетов были получены распределения скорости и давления для чисел Бингама в диапазоне от 1.225 до 17.01 и чисел Рейнольдса от 71.2 до 740.8. Увеличение числа Рейнольдса до Re = 740.8 и снижение числа Бингама до Bn = 1.225 приводят к сокращению области с максимальными скоростями и изменению входного распределения аксиальной скорости. Радиальные профили аксиальной скорости остаются одинаковыми на всех поперечных сечениях от z/R = 10 до z/R = 40, что указывает на установление стационарного режима течения вязкопластичной жидкости, в котором образуется постоянное ядро скорости в поперечном сечении трубы.

Ключевые слова: течение вязкопластичной жидкости, аппарат эффективной молекулярной вязкости, предел текучести, модель Бингем-Папанастасиу.

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