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## Forecasting the involvement of residual reserves in the development of a late-stage field

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<p>Received: September 21, 2024 Peer reviewed: October 24, 2024 Accepted: November 6, 2024</p>	<p><b>ANNOTATION</b></p> <p>In the current context of the energy industry, effective management of residual reserves of fields at late stages of development is becoming a matter of critical importance. Residual oil reserves play a key role not only in ensuring energy security but also in the formation of economic sustainability of regions and countries. One of the main aspects of residual reserves management is forecasting their involvement in development at the later stages of the field life cycle. The presence of old fields, for which the construction of a GHDM is inappropriate, determines the use of various analytical and mathematical models in the analysis and design of development. The variety of such models is great, which allows them to be applied to various fields and at various stages of development. Of the many numerical models, we can highlight those that are distinguished by: ease of use; absence of complex physical and technological formulas; wide applicability for various categories of deposits; as well as the absence of the need for a detailed study of the geological and physical characteristics of the deposit. In our case, we are talking about displacement characteristics (hereinafter referred to as DC), which are a powerful data analysis tool that makes it possible to identify patterns and trends in changes in residual reserves. The use of statistical models allows us not only to assess the current state of residual reserves but also to predict their behavior in the future, which is a key element of effective production management.</p> <p><b>Keywords:</b> field, well, displacement characteristics, model, production, production analysis.</p>
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### Introduction

Forecasting the involved oil reserves is based on the assessment of the available oil resources in the field and their potential production. This includes the

analysis of the geological structure of the field, the properties of the reservoir, the hydrodynamic conditions, and the technological possibilities of production. Knowledge of oil reserves is a key element for making decisions on field development

planning, choosing the optimal technologies and production methods, and assessing the economic efficiency of the project. Thus, the determination of oil reserves is closely related to the forecast of the reserves involved and plays an important role in the management of production and development of oil fields. Oil reserves are the volume of oil that can be extracted from a field using current technologies and production methods, subject to economically favorable conditions. This concept includes an assessment of the amount of oil that is physically present in the geological structure (deposit) and also takes into account the technical and economic feasibility of its extraction. The determination of oil reserves includes a comprehensive assessment of geological, hydrodynamic, engineering and economic factors.

The topic of forecasting the involvement of residual reserves in the development of a late-stage field using displacement characteristics is relevant for oil-producing companies, as it allows for increasing the efficiency of oil production. In this review, we will consider some of the studies and publications devoted to this topic.

The research "Predicting the Residual Reserve Involvement of Late-Stage Fields Using Displacement Characteristics" discusses various methods for predicting the residual reserves of late-stage fields using displacement characteristics. The authors propose using hydrodynamic modeling and machine learning methods to estimate the residual reserve involvement. The study was conducted based on field data in China and showed high accuracy in predicting the residual reserve involvement [[1], [2], [3]].

The research "Determination of Residual Reserves of Fields at Late Stages of Development Using Displacement Characteristics". Methods for determining residual reserves of fields at late stages of development using displacement characteristics. The authors propose to use the modeling of the displacement of the injected fluid and the assessment of pressure changes in the late stages of development to predict the remaining reserves. The study was conducted based on the data of the field in Azerbaijan and showed good accuracy in predicting the remaining reserves [4].

The book is a valuable resource for studying the features of the involvement of residual oil reserves. Residual oil reserves are a significant portion of oil reserves that cannot be extracted by standard production methods. Lysenko's book analyzes in

detail various methods and technologies for the involvement of residual oil reserves, such as hot crushing technology, hydrothermal decomposition, steam injection, explosive methods, and many others [5].

The book is an important source of information on the involvement of residual oil reserves in the production process. The authors examine various methods of designing oil field development taking into account the involvement of residual reserves. The book describes in detail the features of the geological structures of oil fields and methods for determining their parameters [6].

Thus, the use of displacement characteristics to predict the involvement of residual reserves in the development of a late-stage field is a promising direction and can lead to increased efficiency of oil production.

## Research methods

In the article, for the analysis of the reserves involved in development under the development system that has developed over a long period, some of the most common displacement characteristics presented in Table 1 were used. The indicated displacement characteristics show the closest to the actual result when developing a field in an active water drive mode [7].

The displacement characteristic method based on the dependence of the cumulative production ( $Q_{cum}$ ) on the natural logarithm of the water-oil ratio (WOR) is one of the methods for analyzing and evaluating the efficiency of production in oil fields. This method is based on the fact that WOR is one of the key parameters that determine the amount of oil that can be extracted from the reservoir. The natural logarithm ( $\ln$ ) of WOR is used to create a linear dependence, which simplifies the analysis and interpretation of the results. The basic steps of the  $\ln(WOR)$  displacement characterization method include Data collection: Oil production data are collected at various stages of field development, including oil and water production volumes;  $\ln(WOR)$  calculation: For each time interval or production stage, the natural logarithm of the WOR is calculated based on the oil and water production [8]; Plotting a graph: A graph is plotted with  $\ln(WOR)$  on the  $\{x\}$  axis and total oil production on the  $\{y\}$  axis. This allows us to visualize the relationship between these parameters.

**Table 1** - Displacement characteristics used

Name of the displacement characteristic	Equation
Method of Nazarov S.N. and Sipachev N.V. [11]	$V_{o.ext}(f_{w.exp.}) = \frac{1}{a} \left( 1 - \sqrt{\frac{(b-1)(1-f_w)}{f_w}} \right)$
Dependence of total production on Ln(WOR)	$Q_{inv} = (w-b)/a$
Dependence of specific production on total	$Q_{inv} = (q_{min}-b)/a$
Method of Sazonov B.F. [12]	$V_{o.ext}(f_{w.exp.}) = \frac{1}{a} \ln \left( \frac{1}{(1-f_w)ae^b} \right)$
Method of Maximov M.I. [10]	$V_{o.ext}(f_{w.exp.}) = \frac{1}{a} \ln \left( \frac{f_w}{(1-f_w)ae^b} \right)$
Method of Kambarov G.S. [13]	$V_{o.ext}(f_{w.exp.}) = a f_w$

Analysis and interpretation: The resulting graph is analyzed to identify trends and patterns. In particular, the shape of the graph, the slope of the line, and other characteristics that may indicate production characteristics and development efficiency are analyzed.

Use of results: The results can be used to forecast production based on current WOR data, as well as to optimize production strategies and make decisions about further field development.

This method allows us to evaluate the influence of the water-oil factor on the production process and to determine the optimal production methods for oil recovery.

The reserves involved in development according to this method are determined by the following formula:

$$Q_{inv} = (w-b)/a \tag{1}$$

where  $w$  is the maximum water cut at the end of development (98%);

$a, b$  are empirical coefficients that are determined by the linear section of the development history;

$a$  is the dependence of  $\ln(WOR)$  on  $Q_{cum}$ ;

$b$  is  $(\ln(WOR) - Q_{cum}) * a$ ,  $\ln(WOR)$  and  $Q_{cum}$  are average values for the selected period..

The next method of displacement characterization is an engineering method of analyzing oil production data, which is based on the assumption that the production rate decreases over time according to some law. This method is widely used in the oil and gas industry to predict future production, estimate remaining reserves, and optimize field development strategies [9].

The dependence of specific production ( $q_{spec}$ ) on total production is one form of displacement characteristic that describes the change in the rate of production (specific production) of oil over time as a function of total production. This is expressed in an equation of the following form:

$$Q_{inv} = (q_{min}-b)/a \tag{2}$$

where  $q_{min}$  is the minimum flow rate (0.5 t/day);  $a, b$  are empirical coefficients that are determined by the linear section of the development history;

$a$  is the dependence of  $q_{spec}$  on  $Q_{cum}$ ;

$b$  is  $(q_{spec} - Q_{cum}) * a$ ,  $q_{spec}$  and  $Q_{cum}$  are average values for the selected period.

This equation describes the exponential decline in specific production with increasing total production. The coefficient  $b$  is often called the decline factor and is a measure of the rate of decline in production.

This displacement characterization method allows the parameters of this equation to be estimated from historical production data and used to predict future production. It is a useful tool for field development planning and determining optimal production strategies.

*Method of Maximov M.I. (1959)*

A study of the process of oil substitution by water on a reservoir model, which was presented as a pipe filled with sand. As a result of this study, an empirical relationship was found between the volume of total water production and the volume of total oil production. Based on the analysis, a method based on the close relationship between the volume

of total oil and water production is especially evident at the final stage of oil deposit development [[10], [11]].

According to this method, the relationship between the volume of total water production ( $V_w$ ) and the volume of total oil production ( $V_o$ ) is described by the exponential function equation:

$$V_w = ba^{V_o} \quad (3)$$

where  $V_w$  is the total water production under reservoir conditions;

$V_o$  is the total oil production under reservoir conditions;

$a, b$  are empirical coefficients.

This equation of dependence  $V_w = f(V_o)$  when moving to a linear form is represented by the dependence

$$\ln V_w = aV_o + b \quad (4)$$

where  $a = \ln \alpha$ , and  $b = \ln \beta$  are empirical coefficients.

According to this method, the dependence is constructed using semi-logarithmic coordinates  $Y = \ln V_w$ ,  $X = V_o$ . The resulting dependence on the final section is a straight line with an angular coefficient  $a = \ln \alpha$  and a segment plotted on the ordinate axis  $\{y\} - b = \ln \beta$ . A linear segment is selected from the constructed dependence, according to which it is necessary to determine the empirical coefficients  $a$  and  $b$ . It should be taken into account that this dependence approaches a linear form on the final section of the curve. Therefore, for the most complete reflection of this dependence  $V_w = f(V_o)$ , the values corresponding to this final section are selected. For the given data, the coefficients of linear approximation  $a$  and  $b$  are found using the least squares method. Maksimov M.I. asserts that forcing liquid extraction and pumping displacement agents into the formation (specifically when pumping water) do not have a significant effect on the straightness of the final section of this DC method. From this, it follows that this method can be used to predict the involved reserves in deposits with a reservoir pressure maintenance system by pumping water [12].

Let's take a closer look at the capabilities of the method and its potential.

Let's reduce the equation to the form:

$$V_l = V_o + ba^{V_o} \quad (5)$$

Differentiating for time, we obtain:

$$\frac{dV_l}{dt} = \frac{dV_o}{dt} + \frac{d(ba^{V_o})}{dV_o} \frac{dV_o}{dt} \quad (6)$$

$$\frac{dV_l}{dt} = (1 + ba^{V_o} \ln a) \frac{dV_o}{dt} \quad (7)$$

Since the change in oil volume relative to the change in liquid volume is determined by the function  $f_o$ , the remaining oil reserves under reservoir conditions can be calculated by setting the limiting value of the oil content  $f_o$

$$1 = f_o + f_o a^{V_o} b \ln a \quad (8)$$

$$\ln \left( \frac{1 - f_o}{f_o b \ln a} \right) = \ln (a^{V_o}) \quad (9)$$

$$V_o = \frac{1}{\ln a} \ln \left( \frac{1 - f_o}{f_o b \ln a} \right) \quad (10)$$

Therefore, the involved oil reserves for the established value of  $f_o$  will be determined by the formula:

$$V_{o.exp} (f_{w.exp}) = \frac{1}{a} \ln \left( \frac{1 - f_o}{f_o a e^b} \right) \quad (11)$$

Thus, the involved oil reserves for a certain limiting value of water cut  $f_w$  will be calculated according to the following expression:

$$V_{o.exp} (f_{w.exp}) = \frac{1}{a} \ln \left( \frac{f_w}{(1 - f_w) a e^b} \right) \quad (12)$$

where  $a$  and  $b$  are the linear approximation coefficients, which are determined using the least squares method.

The predicted total volume of water production, corresponding to the value  $V_o(f_{o.exp})$  or  $V_o(f_{w.exp})$ , can be calculated as

$$V_w = e^{aV_o + b} \quad (13)$$

The predicted total liquid production volume corresponding to the value  $V_o(f_{o.exp})$  or  $V_o(f_{w.exp})$ , can be calculated as

$$V_l = V_o + e^{aV_o + b} \quad (14)$$

*Method of Sazonov B.F. (1973)*

The method assumes that there is a relationship between the total oil and liquid production, which is especially clear at the final stage of oil reservoir development. This method assumes that the relationship between the volume of liquid production ( $V_w$ ) and the volume of oil production ( $V_o$ ) is described by an exponential function equation [13].

$$V_l = ba^{V_o} \tag{15}$$

where  $V_l$  is the total liquid production under reservoir conditions;

$V_o$  is the total oil production under reservoir conditions;

$\alpha, \beta$  are empirical coefficients.

This equation of dependence  $V_w = f(V_o)$  when moving to a linear form is represented by the dependence

$$\ln V_l = aV_o + b \tag{16}$$

where  $a = \ln \alpha$ , and  $b = \ln \beta$  are empirical coefficients.

The dependence is constructed in semi-logarithmic coordinates  $Y = \ln V_l, X = V_o$ . The obtained dependence reveals the range on which it is necessary to determine the empirical coefficients  $a$  and  $b$ . The obtained dependence approaches a linear form on the final section, and the values from this section are used to determine the coefficients. For the given data, the coefficients of linear approximation  $a$  and  $b$  are found using the least squares method [14].

Differentiating the equation concerning time, we obtain

$$\frac{dV_l}{dt} = \frac{d(ba^{V_o})}{dt} = \frac{d(ba^{V_o})}{dV_o} \frac{dV_o}{dt} \tag{17}$$

$$\frac{dV_l}{dt} = ba^{V_o} \ln a \frac{dV_o}{dt} \tag{18}$$

Since the ratio of the change in oil volume to the change in liquid volume is a function of the oil content, it is possible to determine the oil reserves at reservoir conditions by setting a limiting value of the oil content  $f_o$ .

$$1 = f_o a^{V_o} b \ln a \tag{19}$$

$$\ln \left( \frac{1}{f_o b \ln a} \right) = \ln(a^{V_o}) \tag{20}$$

$$V_o = \frac{1}{\ln a} \ln \left( \frac{1}{f_o b \ln a} \right) \tag{21}$$

In this case, the oil reserves in reservoir conditions for a given value of oil content  $f_o$  will be calculated using the following expression

$$V_{o,ext}(f_{w,exp}) = \frac{1}{a} \ln \left( \frac{1}{f_o a e^b} \right) \tag{22}$$

Then the oil reserves for the established maximum water cut value  $f_w$  will be determined by the following expression.

$$V_{o,ext}(f_{w,exp}) = \frac{1}{a} \ln \left( \frac{1}{(1 - f_w) a e^b} \right) \tag{23}$$

where  $a$  and  $b$  are the linear approximation coefficients, which are determined using the least squares method.

As a result, it is possible to determine the predicted total liquid production corresponding to the values at the given expected water cut

$V_o(f_{o,exp})$  or  $V_o(f_{w,exp})$ , as:

$$V_l = ea^{V_o} \tag{24}$$

The methods of Maksimov M.I. (1959) and Sazonov B.F. (1972) have similarities in approach and therefore are often used in the same areas. However, the method of Sazonov B.F. (1972) in some cases shows more resistance to changes in the system of development of objects than the method of Maksimov M.I. (1959) [[15], [16], [17]].

*Method of Kambarov G.S. (1974)*

This method, developed by G.S. Kambarov, is similar to the method of Pirverdyan A.M. (1970), but it is based not on the inverse-square, but on a simpler inverse relationship between  $V_o$  and  $V_l$ . Research conducted by the author of this method revealed a relationship between the total production of oil and the following type of liquid [[16], [17]].

$$V_o = a + \frac{b}{V_l} \tag{25}$$

where  $V_l$  is the total liquid production under reservoir conditions;

$V_o$  is the total oil production under reservoir conditions;

$a, b$  are empirical coefficients.

The equation  $V_o = f(V_l)$  can be applied in two variants: the first is the original equation proposed by the author, and the second is the equation

transformed into a linear form. When transformed into a linear form, the following dependence can be presented:

$$V_o V_l = a V_l + b \tag{26}$$

Calculations according to this method are performed as follows. The dependence is constructed in coordinates  $Y = V_o$ ,  $X = 1/V_l$  for the basic method, and in coordinates  $Y = V_o V_l$ ,  $X = V_l$  for the modified method. The obtained dependence reveals the range on which it is necessary to determine the empirical coefficients  $a$  and  $b$ . For the given data, the coefficients of linear approximation  $a$  and  $b$  are found using the least squares method [16].

Let's study the DC model of Kambarov G.S. in more detail. Let's reduce the equation to the form:

$$V_l = a + \frac{b}{V_o - a} \tag{27}$$

Differentiating concerning time, we obtain:

$$\frac{dV_l}{dt} = \frac{d}{dt} \left( \frac{b}{V_o - a} \right) = \frac{d}{dV_o} \left( \frac{b}{V_o - a} \right) \frac{dV_o}{dt} \tag{28}$$

where  $u = V_o - a$ .

It follows from this,

$$\frac{dV_l}{dt} = (-bu^{-2}) \frac{dV_o}{dt} = -b(V_o - a)^{-2} \frac{dV_o}{dt} \tag{29}$$

$$\frac{dV_l}{dt} = \frac{-b}{(V_o - a)^2} \frac{dV_o}{dt} \tag{30}$$

Since the change in oil volume relative to liquid volume is a function of oil content, it is possible to determine the oil reserves in reservoir conditions by setting a limiting value for oil content.

$$1 = \frac{-bf_o}{(V_o - a)^2} \tag{31}$$

Therefore, the volume of oil reserves for a given oil content value will be determined by this formula:

$$V_o = a - \sqrt{-bf_o} \tag{32}$$

Thus, the oil reserves for a given water content limit will be calculated according to this expression

$$V_o = a - \sqrt{bf_o - b} \tag{33}$$

where  $a$  and  $b$  are the linear approximation coefficients, which are determined using the least squares method.

For the models we are considering, the value  $a$  characterizes the maximum possible recoverable oil reserves with infinite formation flushing. This follows from the equation: as the oil content tends to zero  $f_o \rightarrow 0$ , the value  $V_{o,ext} \rightarrow a$ , then

$$V_{o,ext,max} = a \tag{34}$$

The predicted total liquid recovery corresponding to the volume  $V_o$  can be defined as:

$$V_l = \frac{b}{V_o - a} \tag{35}$$

The predicted total water recovery corresponding to the volume  $V_o$  can be defined as:

$$V_w = \frac{b}{V_o - a} - V_o \tag{36}$$

*The method of Nazarov S.N. and Sipachev N.V. (1972) describes a direct dependence of the growth of the water-oil ratio (WOR) on the increase in water production with an increase in the percentage of water in the extracted product [[10], [11], [12], [13]]. The higher the total water-oil ratio and the more stable and uniform the development of the studied object is, the more appropriate it is to use the specified methods [[18] [19]].*

The equation for this method is written

$$\text{as: } V_{o,ext}(f_{w,mar.}) = \frac{1}{a} \left( 1 - \sqrt{\frac{(b-1)(1-f_w)}{f_w}} \right) \tag{37}$$

where  $a$  and  $b$  are the linear approximation coefficients, which are determined using the least squares method.

To find the coefficients  $a$  and  $b$ , the last points are used in all cases, allowing the methods to be reduced to a single approximation period. The number of points is chosen arbitrarily (at least 5), mainly from the condition that the dependence section could be linearly approximated with a high degree of reliability ( $R^2$ ), i.e. determine the equation of the linear dependence between  $\{x\}$  and  $\{y\}$ .

In order to track the migration of fluids in the reservoir, experimental studies are conducted on physical models of the productive formation to determine the coefficient of oil displacement by water, relative phase permeability, capillary pressure and wettability of rocks in the X field.

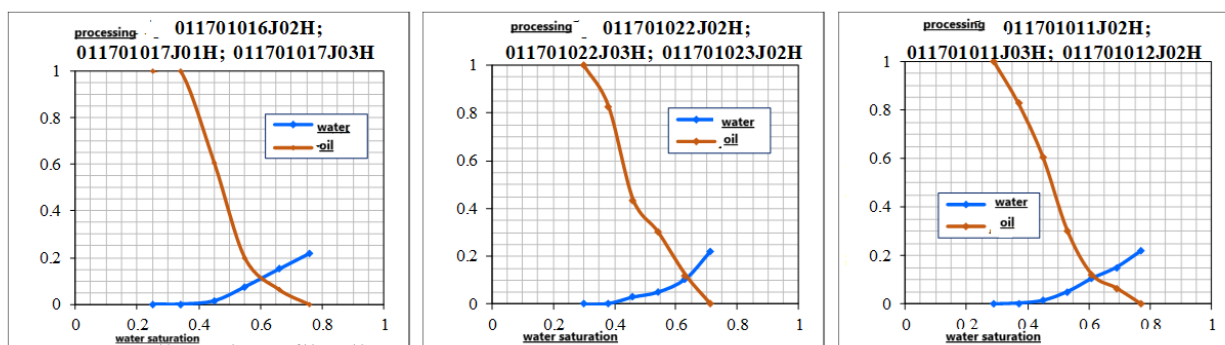
This subsection analyzes special well studies. The following is an analysis of the experimental results. Table 2 presents the set of special studies and the number of samples used.

**Table 2** - Types of special studies conducted

Type of Research	Quantity
Capillary pressure curves. sample	4
Relative phase permeability in the oil-water system. model/sample	3/9
Oil displacement coefficient by water. model/sample	4/11

**Table 3** - Relative permeability in the water-oil system

Model number	Depth, m	Core data		Results of the experiment				
		Porosity, %	Gas permeability, mD	Residual water saturation Swir, units	Residual oil saturation Sor, units	Relative permeability to water at Sor, fractions of units.	Relative permeability for oil at Swir, units.	Oil displacement coefficient by water, units.
1	855.38; 856.17; 856.87	37.99	1946.67	0.246	0.169	0.219	1	0.78
2	887.54; 887.79; 888.59	37.97	1846.67	0.297	0.23	0.219	1	0.67
3	850.42; 850.74; 851.58	36.05	1586.67	0.287	0.228	0.151	1	0.68



**Figure 1** – Relative permeability curves for oil and water

Determination of relative phase permeabilities in the oil-water system. The experiment was conducted on 3 models of 9 core samples. The final results of the experiment are presented in Table 3. [17].

Based on the results of determining the relative permeabilities, the residual water saturation varies within the range of 0.246-0.297 units, averaging 0.277 units. The relative permeability curves are shown in Figure 1.

The intersection of the relative phase permeability curves on the samples characterizes the rocks as hydrophilic.

*Determination of the displacement coefficient in the oil-water system.* The oil displacement coefficient was determined for 11 samples, from which 4 reservoir models were assembled. The oil displacement coefficient varies in the range of 0.60÷0.80 fractions of a unit, averaging 0.73 fractions of a unit (Table 4) [[20] [21]].

**Table 4** - Results of determination of the coefficient of oil displacement by water

Model №	Depth, m	Porosity, %	Gas permeability, mD	Residual water saturation, $S_w^{res}$ , units	Residual oil saturation, $S_o^{res}$ , units	Displacement coefficient, $\beta$ , units
4	851.07; 852.44; 859.55	35.49	1626.67	0.263	0.147	0.801
5	853.73; 854.06; 856.6	33.89	1449	0.282	0.209	0.709
8	889.88; 890.38; 891.54	36.24	1176.33	0.251	0.154	0.79
6	850.05; 852.18	31.69	1185	0.27	0.294	0.597

**Table 5** - Characteristics of oil displacement by water in productive formation zones

Name of quantities	Permeability, $10^{-3} \mu\text{m}^2$	Bound water content, units	Initial oil saturation, units.	Residual oil saturation during displacement of oil by the working agent, units.	Oil displacement coefficient, units.	Relative permeability values, units.	
						for the working agent at residual oil saturation	for oil at saturation with bound water
Number of definitions	5	5	5	5	5	2	2
Average value	1558.8	0.269	0.731	0.209	0.713	0.185	1

Table 5 shows the characteristics of oil displacement by water in the zones of the productive formation.

This table uses data on the oil displacement coefficient by water and relative phase permeability.

According to the results of the experiment, the  $S_w^{res}$  values vary from 0.246 to 0.287 fractions of units, the average value is 0.269 fractions of units,  $S_o^{res}$  – from 0.147 to 0.294 fractions of units and is characterized by an average value of 0.209 fractions of units. The values of the oil displacement coefficient by water vary from 0.597 to 0.800 fractions of units, averaging 0.713 fractions of units.

Based on the acquired knowledge of constructing displacement characteristics and analyzing the current state of development for assessing the recoverable oil reserves involved in development, displacement characteristics were constructed using the methods of Nazarov S.N. and Sipachev N.V., the dependence of total production on  $\ln(\text{WOR})$ , the dependence of specific production on total, Maksimova M.I., Sazonova B.F., Kambarova G.S. [[10], [11], [12], [13]] The calculation results for the objects are presented in Table 6 and Figures 2-4. These methods assume the determination of the reserves involved by the final stage of development under the existing system.



**Table 6** - Estimated involved oil reserves and recovery factors by sites

Indicators	Sites			Total	
	I	II	III		
Approved OGR, thousand tons	24471	5677	1627	31775	
Approved LWR, thousand tons	12317	3323	1032	16672	
Approved ORF, units	0.503	0.585	0.634	0.525	
Cumulative production, thousand tons	11842	3152.7	765.7	15761.3	
Current ORF, units	0.484	0.555	0.471	0.496	
Selection from LWR, %	96.2	94.9	74.2	94.5	
Estimate of recoverable oil reserves, thousand tons	475	170.3	263	911	
Involved reserves, thousands of tons	13446.1	3610.6	1202	18259	
Potential ORF, units	0.549	0.636	0.739	0.575	
Potential estimate of recoverable oil reserves, thousands of tons	1604.1	457.9	433	2495	
Dependence of total production on Ln(WOR)	reserves	13350	3291	1495	18136
	ORF	0.546	0.580	0.919	0.571
Dependence of specific production on total	reserves	12942	3910	977	17829
	ORF	0.529	0.689	0.600	0.561
Methodology of Maximov M.I.	reserves	13640	3662	1249	18552
	ORF	0.557	0.645	0.768	0.584
Methodology of Sazonov B.F.	reserves	13796	3670	1248	18714
	ORF	0.564	0.647	0.767	0.589
Methodology of Kambarova G.S.	reserves	13502.4	3574	1044	3555
	ORF	0.552	0.630	0.641	0.570

### Site I of the X deposit

The main object at the field is Site I, which contains 74% of the initial recoverable oil reserves of the entire field. The total drilled stock of Object I is 177 wells. As of 01.01.2024, the current operating stock for the object is 89 wells, of which 74 are producing and 15 are injection wells. The cumulative oil production is 11,842.9 thousand tons, the production from LWR is 96.1%, and the water cut of the production has reached 90.8%. The current oil recovery factor is 0.484 shares of units, with the approved one being 0.503 shares of units.

Site I is at the final IV stage of development, and the staging of this object, as well as the deposit as a whole, differs in some way from the accepted classical schemes: a very short period of stage II development and a long period of development at the final stage, which has already lasted almost 30 years, while, despite the long time, less than 20% of the accumulated production for the object as a whole has been extracted during the period of stage IV.

A significant period of stable development of the field is also accompanied by the stabilization of the water cut of the extracted products, which creates prerequisites for constructing very reliable

characteristics of displacement and more accurate determination of reserves involved in development. Reserves involved in development today are determined based on the average result of construction for various characteristics. The use of the arithmetic mean value for determining the involved reserves is because similar values were obtained by all methods.

The calculations were complicated by the last few years of development, during which there has been a decrease in water cuts and an increase in annual oil production. Such changes are explained by an increase in the stock of production wells: in the period from 2017 to 2023, 20 new wells were put into operation at the 1st facility (from drilling, conservation, and transfer from other facilities).

The results of constructing displacement characteristics show that the involved reserves amount to 13.5 million tons, which is approximately 10% higher than the approved recoverable reserves. Looking ahead, it can be said that some excess of the projected accumulated oil production over the approved recoverable reserves is also confirmed by the results of calculations of the predicted technological indicators of development.

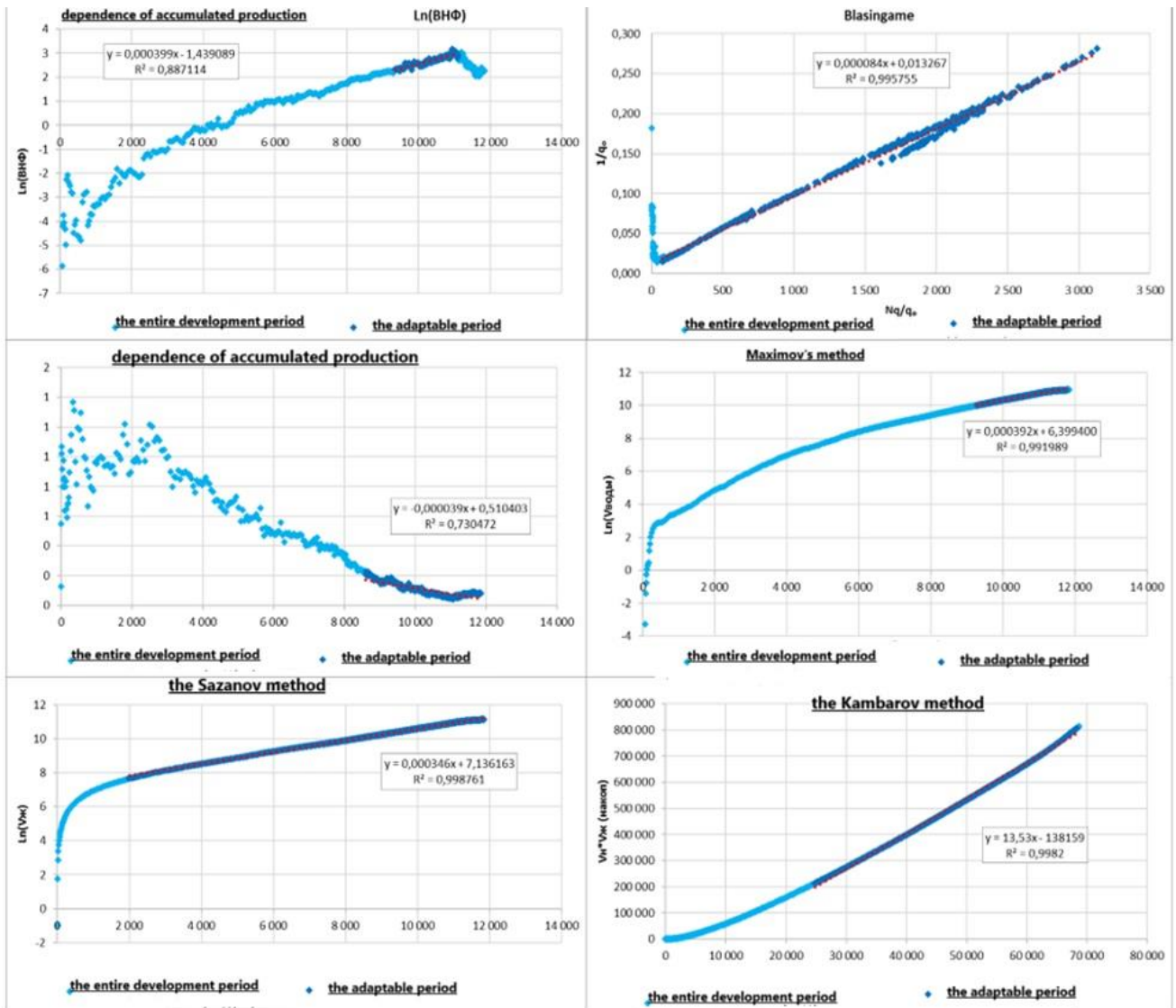


Figure 2- Characteristics of displacement by Site I

To begin with, in order to understand whether it is possible to achieve the obtained involved reserves based on the calculation results for all three objects, a calculation of the base production was performed, i.e. continuation of the development of the field with the existing fund while maintaining the current decline in oil production to the maximum water cut of 98%.

To find the rate of decline for operational sites I and II (for site III it is given above as an example), oil flow rate graphs were constructed and the rate of decline for the last years was determined by the exponential trend. Figures 2 and 3 show the graphs for determining the rate of decline for the historical flow rate for I and II objects.

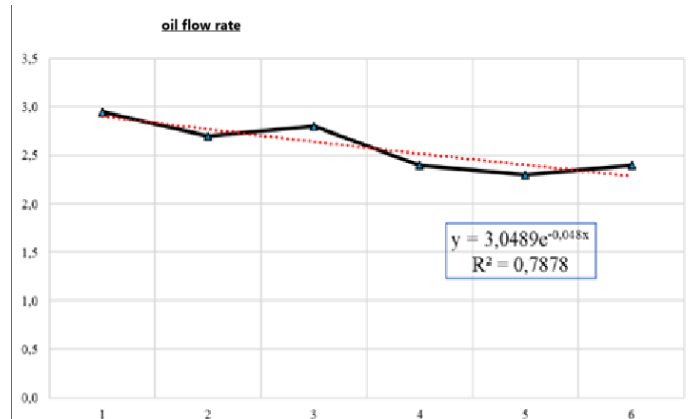
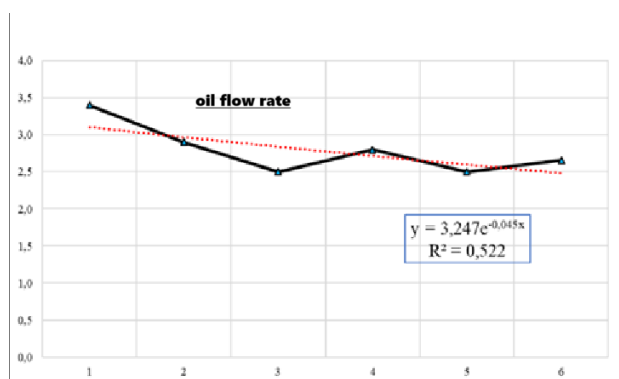


Figure 3 – Determination of the decline rate based on the historical oil flow rate of Site I



**Figure 4** - Determination of the decline rate based on the historical oil flow rate of Site II

The rates of decline in oil production determined from the charts were incorporated into the calculations of the predicted technological indicators. A specially developed template in the

program «Microsoft Office Excel» was used to calculate the technological indicators.

#### I site of the deposit X

According to the results of constructing the displacement characteristics, the involved reserves are higher than the recoverable reserves by 9.2%, which indicates that with the current development system, it is possible to achieve the approved values of the recovery factor for the object. From the presented Table 6 it is also clear that the predicted cumulative production upon reaching the maximum water content (98%) will exceed the recoverable reserves by 5.9% and will amount to 13,043 thousand tons with the approved 12,317 thousand tons, the difference in the calculations is only 3.3%, which, taking into account the long period of development and no less long forecast period, can be considered an error.

**Table 7** - Example of determining the initial flow rate of horizontal well № 303

Initial data for calculating the flow rate of the HW							
$c$	0.05	coefficient when using commercial units					
$\mu$	2.25	liquid viscosity, cP					
$r_{eh}$	200.0	power circuit radius, m					
$r_c$	0.10	wellbore radius, m					
$B_0$	1.12	volumetric coefficient					
$L$	450.00	horizontal wellbore length, m					
$S$	0.00	skin factor					
Calculation							
$Kh$	$Kv$	$h$	$Preservoir$	$P_{bottomhole}$	$a$	$b$	$Q_{liquid}$
2.4	0.241	15	109.3	82	269.53	3.16	<b>19.1</b>
Total							
Predicted initial water cut, %							<b>15.0</b>
Estimated initial oil flow rate, t/day							<b>11.8</b>

**Table 8** - Well commissioning schedule for object I

Years	Site	Activity	Well №	Initial oil flow rate, t/day
2026	III	Input from <b>vertical</b> well drilling	300	9.5
2027	III	Input from <b>vertical</b> well drilling	301	9.0
2028	III	Input from <b>horizontal</b> well drilling	302	11.4
2028	III	Input from <b>horizontal</b> well drilling	303	11.8

To calculate the initial flow rate, the **Joshi formula** was used:

$$Q = \frac{K_h h \Delta P}{\mu B_o \left\{ \ln \left[ \frac{a + \sqrt{a^2 - (L/2)^2}}{L/2} \right] + \frac{\beta h}{L} \ln \frac{\beta h}{(\beta + 1)r_c} + 1 \right\}} \quad (38)$$

$$a = \frac{L}{2} \left[ 0,5 + \sqrt{0,25 + \left( \frac{2r_{eh}}{L} \right)^4} \right]^{0,5} \quad \beta = \sqrt{\frac{K_h}{K_v}}$$

where:

- $\mu$  – fluid viscosity, cP;
- $r_{eh}$  – feed contour radius, m;
- $r_c$  – wellbore radius, m;
- $B$  – volumetric coefficient;
- $L$  – horizontal wellbore length, m;
- $S$  – skin factor;
- $K_h$  – horizontal permeability, mD;
- $K_v$  – vertical permeability, mD;
- $h$  – effective oil-saturated thickness, m;
- $\Delta P$  – depression.

For example, below is an example of determining the oil flow rate of the project horizontal well № 303 using the Joshi formula, presented in Table 7.

Table 8 shows the well drilling schedule according to the recommended option for object III.

As a result of drilling horizontal and vertical wells at the I object and subsequent calculations of technological development indicators, we observe an increase in annual oil production volumes and oil recovery factor (ORF). In particular, ORF increased from 0.580 to 0.720 with a maximum water cut of 98%. These calculations indicate the technological efficiency of the well drilling activities at the I object.

## Conclusion

In today's energy sector, effective management of residual reserves in the late stages of development is becoming critical. Residual oil reserves not only ensure energy security but also contribute to the economic sustainability of regions and countries.

A key aspect of managing these reserves is to predict their involvement in development at later stages of the field life cycle. For older fields, where the creation of hydrodynamic models is impractical, various analytical and mathematical models are used

to analyze and design development. These models are diverse and can be applied to different fields and stages of development.

In this case, the emphasis is on displacement characteristics (DC), which are a powerful tool for analyzing data and identifying patterns in changes in residual reserves. The use of statistical models allows us to assess the current state of residual reserves and predict their behavior in the future, which is a key element for effective production management.

In the course of this article, a comprehensive analytical study of the current state of field development was conducted. The results of the analysis indicate that there is a lag in the development of reserves at site III. Also, in the process of determining the involved reserves, it became clear that with the current development system, achieving the approved recoverable reserves is impossible. In light of the above factors, specific measures were proposed aimed at achieving and ensuring recoverable reserves. The results of the technical and economic assessment showed that due to the implementation of the proposed measures, it was possible to extend the profitable period of field development.

Most of the methods and technologies for influencing the deposit are already known and are actively used in various countries. However, the effective use of these methods depends on the choice of optimal approaches corresponding to a specific deposit. To make such a choice, it is necessary to have sufficient knowledge of the geological and physical characteristics and correctly represent the geological and industrial model.

**Conflict of interest.** The corresponding author declares that there is no conflict of interest.

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## Кен орындарының соңғы сатыдағы қалдық қорларын игеруге қатыстыруды болжау

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### ТҮЙІНДЕМЕ

Энергетика саласының қазіргі кездегі жағдайында игерудің соңғы кезеңдеріндегі кен орындарының қалдық қорларын тиімді басқару өзекті мәселеге айналып отыр. Мұнайдың қалдық қорлары тек энергетикалық қауіпсіздікті қамтамасыз етуге ғана емес, сонымен қатар аймақтар мен мемлекеттердің экономикалық тұрақтылығын қалыптастыруда шешуші рөл атқарады. Қалдық қорларды басқарудың негізгі аспектілерінің бірі олардың кен орнының өмірлік циклінің кейінгі кезеңдерінде игеруге қатысуын болжау болып табылады. Толық геологиялық-гидродинамикалық модель (ГГДМ) құру тиімсіз болып табылатын ескі кен орындарының болуы, талдау мен әзірлеу барысында түрлі аналитикалық және математикалық модельдерді қолдануды талап етеді. Мұндай саналуан модельдер өте көп, бұл оларды әртүрлі салаларда және дамудың әртүрлі кезеңдерінде қолдануға мүмкіндік береді. Көптеген сандық модельдердің мынадай ерекшеліктерін атауға болады, олар: қарапайымдылығы бойынша; күрделі физикалық және технологиялық формулалардың болмауы бойынша; әртүрлі санаттағы кен орындары үшін кеңінен қолданылуы бойынша; және кен орнының геологиялық-физикалық сипаттамаларын егжей-тегжейлі зерттеуді қажет етпеуі бойынша ерекшеленеді. Біздің жағдайда біз қалдық қорлардағы өзгерістердің заңдылықтары мен тенденцияларын анықтауға мүмкіндік беретін қуатты деректерді талдау құралы болып табылатын ығыстыру сипаттамалары (бұдан әрі - ЫС) туралы айтып отырмыз. Статистикалық модельдерді қолдану қалдық қорлардың ағымдағы күйін бағалауға ғана емес, сонымен қатар олардың болашақтағы іс әрекетін болжауға мүмкіндік береді, бұл өндірісті тиімді басқарудың негізгі элементі болып табылады.

**Түйін сөздер:** кен орны, ұңғыма, ығыстыру сипаттамалары, модель, өндіру, өндіруді талдау.

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## Прогнозирование вовлеченности в разработку остаточных запасов месторождения на поздней стадии

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<p>Поступила: 21 сентября 2024 Рецензирование: 24 октября 2024 Принята в печать: 6 ноября 2024</p>	<p><b>АННОТАЦИЯ</b></p> <p>В современном контексте энергетической индустрии, эффективное управление остаточными запасами месторождений на поздних стадиях разработки становится вопросом критической важности. Остаточные запасы нефти играют ключевую роль не только в обеспечении энергетической безопасности, но и в формировании экономической устойчивости регионов и государств. Одним из основных аспектов управления остаточными запасами является прогнозирование их вовлеченности в разработку на поздних этапах жизненного цикла месторождения. Наличие старых месторождений, построение гидродинамической модели по которым нецелесообразно, обуславливает применение различных аналитических и математических моделей при анализе и проектировании разработки. Разнообразие подобных моделей велико, что позволяет применять их к различным месторождениям и на различных стадиях разработки. Из множества числовых моделей можно выделить такие, которые отличаются: простотой использования; отсутствием сложных физических и технологических формул; широкой применимостью для различных категорий месторождений; а также отсутствием необходимости детальной изученности геолого-физической характеристики месторождения. Речь в нашем случае идет о характеристиках вытеснения (далее-ХВ), которые представляют собой мощный инструмент анализа данных, который позволяет выявлять закономерности и тенденции в изменениях остаточных запасов. Применение статистических моделей позволяет не только оценить текущее состояние остаточных запасов, но и прогнозировать их поведение в будущем, что является ключевым элементом эффективного управления добычей.</p>
	<p><b>Ключевые слова:</b> месторождение, скважина, характеристики вытеснения, модель, добыча, анализ добычи.</p>
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