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# A mathematical model for determining the influence of factors on the stability of pillars and cameras

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**Abstract.** This article presents the results of the research in determining the parameters of the stress-strain state of the rock mass around pillars, cameras and factors, which affect their stability. The vertical section of cirque around the ledges is considered as well. A flat deformed rectangular plane is divided by triangle grid with a corresponding boundary. Boundary conditions are chosen as a calculation scheme. A multidimensional mathematical model of the pit sides stability was obtained from the geological and mining factors. From the obtained formula for a multidimensional model, we can determine the factors that affect the stability of pillars and cameras. The resulting ratio makes it possible to determine the desired value.

**Keywords:** rocks, finite element method, stress-strain state, deformation, stress, development.

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## Introduction

The methodological substantiation of the parameters of mineral extraction is based on studies of the technogenic state, a feasibility study and comparison of alternative development options for the field with various technologically feasible production schemes.

The most important parameters that determine the effectiveness of development systems with an open treatment space are the dimensions of the support and barrier pillars,

which mainly depend on natural factors - reservoir power, depth of development, physical and mechanical properties of rocks and ore mass. The optimal parameters of development systems are established on the basis of formalized laws of their influence on the value of the adopted optimality criterion and are adjusted taking into account restrictions on the stability factor of the stress-strain state (SSS) of the surrounding array and pillars.

The purpose of this study is to determine the SSS of the rock mass around pillars and chambers

depending on the depth of development and the parameters of the adopted technological scheme.

### Research analysis

The results of theoretical studies of the VAT of the rock mass around pillars and chambers are given, depending on the depth of development and the parameters of the adopted technological scheme. For research, we will take an element of an array (square shape) with technologically formed workings, pillars between them.

The targets between the chambers, as an anthropogenic structure, is the main supporting technological element of ore mining systems. Therefore, an important task is to determine the parameters of the stress-strain state of the array, affecting the stability of pillars and treatment chambers and the entire adjacent array. As the design scheme, a rectangular plane is selected, which is in a plane-deformed state, which is divided by a grid of triangular elements, with the corresponding boundary conditions (Figure 1). The problem is solved by the finite element method (FEM) [1].

In the design scheme,  $q = \gamma H$  ( $\gamma$  is the bulk weight,  $H$  is the thickness of the overlying rock mass) is the load acting on the boundary BC; At the boundaries of AB and CD there is no horizontal, vertical displacement at AD. To process studies to determine the SSS of a rock mass, in particular, to determine the stable sizes of supporting pillars and chambers when developing shallow deposits, an unconventional method is used for constructing multidimensional mathematical models [2]. The methodological approach is reduced to the sequential implementation of the following steps:

1. Definition of the purpose of modelling
2. Planning an experiment
3. Conducting an experiment
4. Analysis of the simulation results.

Experiment planning involves:

1. The choice of arguments and objective functions
2. Determining the range of changes in the arguments in the experiments
3. The establishment of the number of levels of arguments and their specific values in the experiments
4. Establishing a combination of argument values

in each experiment in accordance with the recommended experiment planning matrix. In our case, the planning matrix was adopted with the number of arguments  $m = 6$  and the number of levels  $n = 5$ , which fully reflects the completeness of the research starting points. In order to obtain a mathematical model of the type  $y = f(x_1, x_2, x_3, x_4, x_5,$

$x_6)$ , 25 variants of the SSS array were investigated. Assume that the tensile stress is dominant. Then here  $y$  is the maximum main tensile stress:  $x_1 = \gamma H$ ,  $x_2 = b_2$ ,  $x_3 = b_1$ ,  $x_4 = h_2$ ,  $x_5 = E_b$ ,  $x_6 = h_1$ .

In each variant, the problem of determining the SSS of the FEM array was solved. As a result of calculations, vertical and horizontal displacements of nodes, vertical and horizontal normal, as well as tangential and main stresses in the elements.

Figure 2 shows the vertical stress isolines for one of the options, i.e. with the modulus of elasticity of the host rocks,  $E_b = 9.6 \cdot 10^4$  MPa,  $\gamma H = 15$  MPa,  $\sigma_{adm} = 20$  MPa - permissible tensile stress of the rocks,  $b_2 = 11$  m. - height of the pillar,  $h_1 = 24$  m - width of the barrier pillar,  $h_2 = 7$  m the width of the interchamber pillars,  $b_1 = 7$  m is the width of the chamber. In all cases, the modulus of elasticity of ore  $E_{ruda} = 8 \cdot 10^4$  MPa. As can be seen from the figure, zones of development of tensile stresses are observed in the roof and soil of the chambers.

When solving the flat FEM problem, the parameters (technological and mining-geological factors) varied within the following limits:

$b_1 = 4 \div 16$  (m) - chamber width with an interval of 3 m;

$b_2 = 3 \div 11$  (m) - removable power (pillar height) with an interval of 2 m;

$h_1 = 15 \div 27$  (m) - the width of the barrier pillar with an interval of 2 m;

$h_2 = 5 \div 9$  (m) - the width of the inter-chamber pillars with an interval of 1 m;

$\gamma H = 3.75 \div 18.75$  (MPa) - load with an interval of 3.75 MPa;

$E_b = 3.2 \cdot 10^4 \div 9.6 \cdot 10^4$  (MPa) (MPa) - modulus of elasticity with an interval of  $1.6 \cdot 10^4$

According to the above program, a mathematical model is obtained that takes into account the complex of natural and technogenic factors

$$\sigma_1^{\max} = f(x_1, x_2, x_3, x_4, x_5, x_6).$$

The maximum main voltage is selected as the function  $Y$ . And by processing the data the following dependencies of the function on the arguments are obtained:

$$\sigma_1^{\max} = A_1 * \gamma H * B_1 + C_1;$$

$$\sigma_1^{\max} = A_2 * \exp(B_2 * b_1) + C_2;$$

$$\sigma_1^{\max} = A_3 + B_3 / h_1;$$

$$\sigma_1^{\max} = A_4 * \exp(B_4 * E_b);$$

$$\sigma_1^{\max} = A_5 * b_2 ** 2 + B_5 * b_2 + C_5;$$

$$\sigma_1^{\max} = A_6 * h_2 ** (B_6) + C_6,$$

where  $A_1, B_1, C_1, A_2, B_2, C_2, A_3, B_3, A_4, B_4, A_5, B_5, C_5, A_6, B_6, C_6$  are constants.

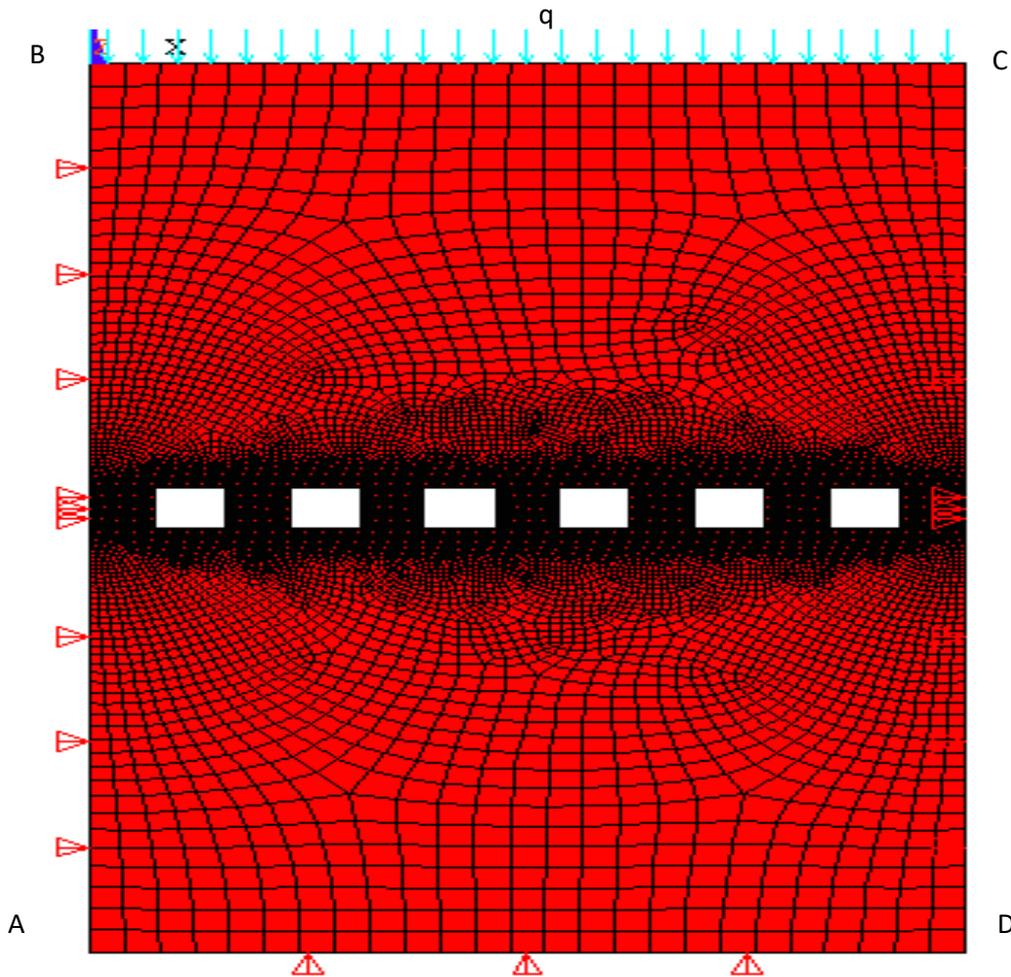


Figure 1 – Design scheme of the problem, adequate field conditions

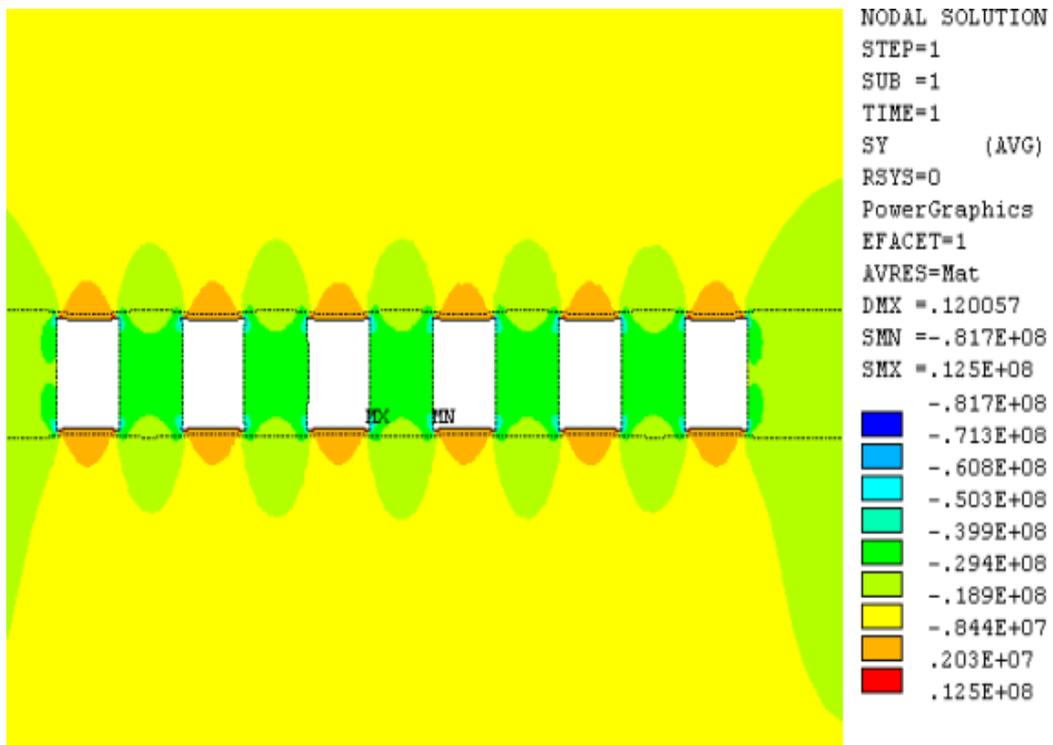


Figure 2 – Isolines of vertical stresses

The correlation coefficient  $R = 0.965$  indicates the tightness of the connection, and the generalized equation has the following form:

$$\sigma_1^{\max} = Y(\gamma H) * Y(b_1) * Y(E_B) * Y(b_2) * Y(h_1) + Y(h_2) \quad (1)$$

According to the formula (1) obtained for a multidimensional model, one can analyze a complex of factors affecting the stability of pillars and chambers. According to this dependence, with known data, the desired value is determined from the following conditions of rock strength:  $\sigma_1^{\max} \leq \sigma_{adm}^p$ , where  $\sigma_{adm}^p$  is the allowable tensile stress.

Testing of the methodology for the changing conditions of the ore deposits of Zhezkazgan is shown in Figures 3-5. At the same time, the dependence of the pillar width on various factors reflecting the main structural elements of the development technology was investigated. When one of the factors changed, the values of the others were fixed. From the results (graphs) of calculations, it follows as:

- a clear (double) effect on the parameter of the pillar is exerted by the power of the deposit;
- the nature of the pressure (density, depth) has the same picture, but in absolute value they qualitatively differ;
- the limit value of the width of the chambers is  $b_1 = 10-18$  m., Above which the size of the pillar sharply increases.

The left pillars, the open roof-soil of the panels (chambers) are “in operation” for a long time (more than 1 year), therefore, the conditions for the development of ore deposits can be attributed to a viscoelastic medium with small deformations in time of structural elements of development systems. To solve viscoelastic problems with small strains, an effective calculation method using variable modules

has been developed. It is known [3] that the solution of linearly-hereditary creep comes down to the replacement of elastic ones. The work uses a temporary operator of the following form:

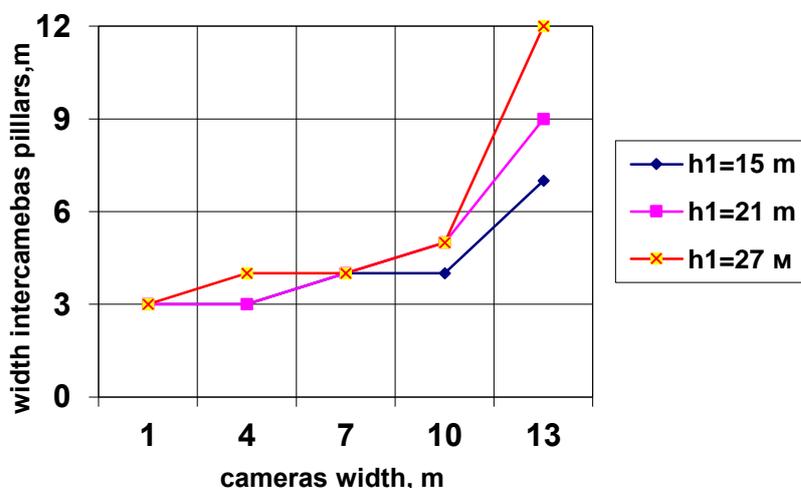
$$E_t = E / (1 + F_t), \quad (2)$$

where  $F_t = \delta t^{1-\alpha} / (1-\alpha)$ ,  $\alpha, \delta$  are creep parameters,  $t$  is time. The objective of the research is to determine the parameters of the stress-strain state (SSS) of the array, affecting the stability of pillars and treatment chambers, taking into account creep constant integral operators. In mining geomechanics, the Abel core  $\delta(t-\tau)^{-\alpha}$  is most often used as the creep core (operator).

When applying the method of variable modules, the solution of the linear hereditary creep problem under constant boundary conditions is reduced to the formulation of the corresponding solution to the problem of the theory of elasticity no longer as an operator, but as a function of time. Formula (1) using the variable module method has the following form:

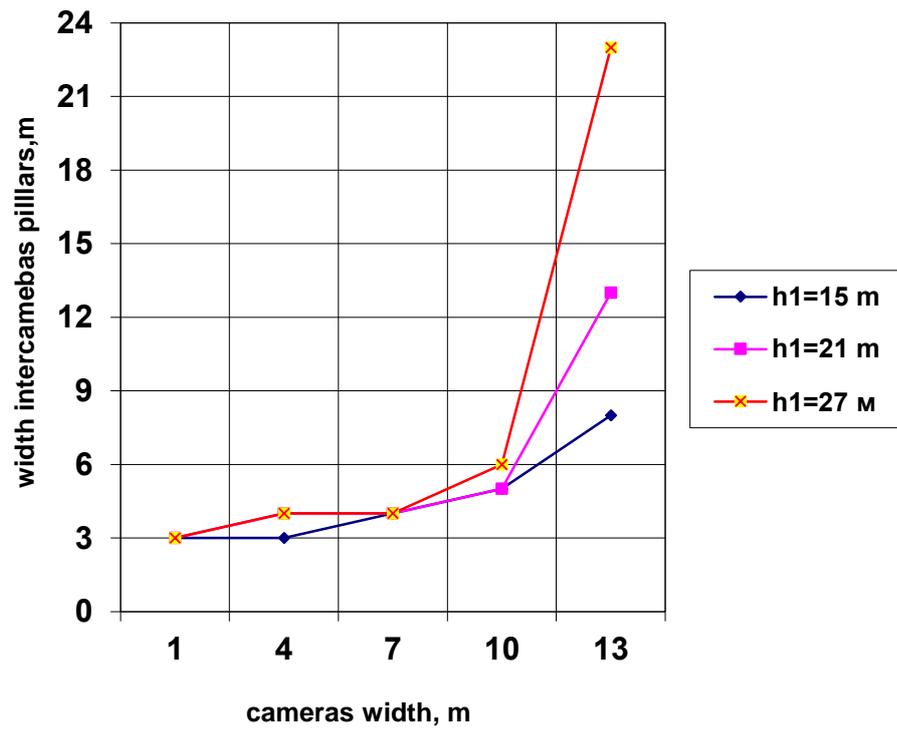
$$\sigma_1^{\max} = (A_1 * \gamma H ** B_1 + C_1) * (A_2 * \exp(B_2 * b_1) + C_2) * (A_4 * \exp(B_4 * E_B / (1 + \delta t^{1-\alpha} / (1-\alpha)))) * (A_5 * b_2 ** 2 + B_5 * b_2 + C_5) * (A_3 + B_3 / h_1) + (A_6 * h_2 ** B_6 + C_6) \quad (3)$$

By the formula (3) obtained for the multidimensional model, one can find a complex of factors affecting the stability of pillars and chambers. According to this dependence, the required value is determined from known values from the following conditions of rock strength:  $\sigma_1^{\max} \leq \sigma_{adm}^p$ , (4) where  $\sigma_{adm}^p$  is the allowable tensile stress.



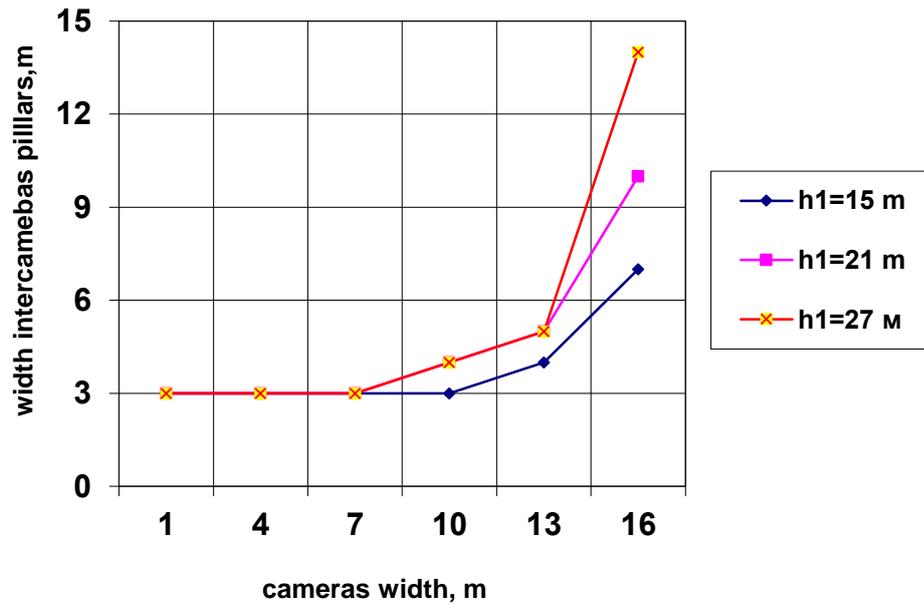
$\gamma H = 7.5$  MPa,  $\sigma_{adm} = 20$  MPa,  $b_2 = 7$  m.,  $E_B = 32000$  MPa

Figure 3 - Dependence of the width of the pillars on the width of the cameras



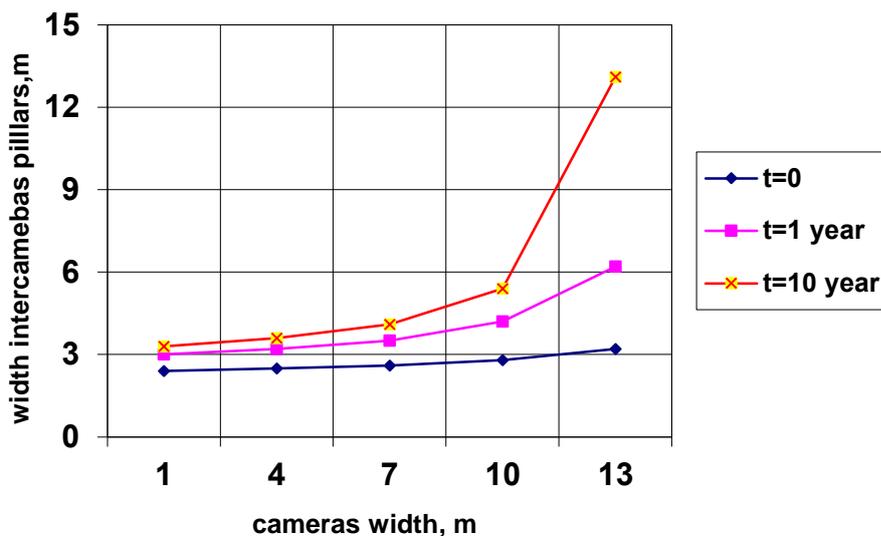
$\gamma H=15 \text{ MPa}$ ,  $\sigma_{adm}=20 \text{ MPa}$ ,  $b_2=7 \text{ m.}$ ,  $E_B=96000 \text{ MPa}$

Figure 4 - Dependences of the width of the pillars on the width of the cameras



$\gamma H=11,25 \text{ МПа}$ ,  $\sigma_{adm}=20 \text{ МПа}$ ,  $b_2=7 \text{ m}$ ,  $E_{B\text{MCH}}=96000 \text{ МПа}$

Figure 5 - Dependence of the width of the pillars on the width of the cameras



$$\gamma H = 7.5 \text{ MPa}, \sigma_{adm} = 20 \text{ MPa}, b_2 = 7 \text{ m}, h_1 = 27 \text{ m}, E_b = 96000 \text{ MPa}$$

Figure 6 - Dependences of the pillar width on  $b_1$ 

So, the mutual influence of the width of the chambers and the pillars changes in time according to the obtained curves (Figure 6). Above we obtained the approbation of the methodology for the changing Zhezkazgan conditions at  $t = 0$ . In this case, the dependence of the pillar width on various factors was studied, which reflected the main structural elements of the development technology.

When one of the factors changed, the values of the others were fixed. Under the same conditions in Fig. 6, for different  $t$  (in days), using the formula

(1), the dependences of the pillar width on the camera width are obtained.

### Conclusions

Thus, the research methodology allows us to establish technologically necessary ratios of elements of development systems (pillar parameters, cameras, panels, etc.) depending on specific conditions. In this case, the optimization of the parameters will affect the level of regulatory losses and ensure stability during the extraction of reserves.

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## Кентірекер мен кенүңгірлердің орнықтылығына әсер ететін факторларды анықтаудың математикалық моделі

Тутанов С. К., Тутанов М. С., Тутанова М. С.

**Түйіндеме.** Бұл мақалада кентірекер мен кенүңгірлер айналасындағы тау жыныстары массивінің кернеулі-деформациялық күйінің параметрлерін және олардың орнықтылығына әсер етуші факторларды анықтау бойынша жүргізілген зерттеулер келтірілген. Кентірекер мен кенүңгірлер маңайындағы массивтің вертикаль қимасы қарастырылған. Есептеу сұлбасы (схемасы) ретінде жазық деформация күйіндегі тік бұрышты жазықтық алынған. Ол сәйкес шекаралары бар үшбұрышты элементтер арқылы бөлінген. Нәтижесінде геологиялық және тау-кен факторларына тәуелді кентірекер мен кенүңгірлер орнықтылығының көп факторлы математикалық моделі алынған. Алынған формула арқылы көп факторлы модель үшін кентірекер мен кенүңгірлер орнықтылығына әсер ететін факторларды табуға болады. Алынған тәуелділік қажетті мәнді анықтауға мүмкіндік береді.

**Түйін сөздер:** тау жыныстары, акыргы элементтер әдісі, кернеулі-деформациялық күй, деформация, кернеу, даму.

### Математическая модель определения факторов, влияющих на устойчивость целиков и камер

Тутанов С. К., Тутанов М. С., Тутанова М. С.

**Аннотация.** В статье приводятся исследования по определению параметров напряженно-деформированного состояния массива горных пород вокруг целиков и камер, а также факторов, влияющих на их устойчивость. Рассматривается вертикальное сечение массива вокруг целиков и камер. В качестве расчётной схемы выбрана прямоугольная плоскость, находящаяся в плоско деформированном состоянии и которая разбивается сеткой треугольных элементов, с соответствующими граничными условиями. Получена многофакторная математическая модель устойчивости целиков и камер от горногеологических и горнотехнических факторов. По формуле, полученной для многомерной модели можно найти комплекс факторов, влияющих на устойчивость целиков и камер. Полученная зависимость дает возможность определить по известным значениям факторов искомую величину.

**Ключевые слова:** горные породы, метод конечных элементов, напряженно-деформированное состояние, деформация, напряжение, развитие.

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